Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 5

Due Date: 14:00 Wednesday, March 18.

Explain your answers! Problems marked ($\star$) are bonus ones.

5.1. Use the Gram-Schmidt procedure to

(a) construct an orthonormal set of vectors from

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 3 \\ 3 \\ 1 \\ -3 \end{pmatrix} \qquad v_4 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

(b) find an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by

$$v_1 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \qquad \text{and} \qquad v_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

5.2. Let $A = (a_{ij})$ be an orthogonal (or unitary) matrix. Show that $|a_{ij}| \leq 1$ for any i, j.

5.3. Find the Jordan normal form and the associated basis of the following matrices:

$$(a)\ \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \qquad (b)\ \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix} \qquad (c)\ \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 3 \end{pmatrix} \qquad (\star)\ \begin{pmatrix} 3 & -1 & 1 & 7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix}$$

5.4. Compute

$$(a)\ \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}^{15} \qquad (b)\ \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}^{20}$$

5.5. ($\star$) Find eigenvalues, eigenvectors, generalized eigenspaces and Jordan normal form of the differentiation operator on the space of polynomials of degree at most n.