

## ESM 2B, Homework 7

**Due Date:** 13:00 Tuesday, April 14.

Explain your answers! Problems marked (★) are bonus ones.

**7.1.** Write down Fourier series for the following functions from  $L^2([-\pi, \pi])$ .

- (a)  $f(x) = |x|$ ;
- (b)  $f(x) = \cos^2(2x)$ ;
- (c)  $f(x) = \sin^2(x)$ ;
- (d)  $f(x) = \cos^2(x) - x$ .

**7.2.** Find the Fourier series of the function  $f(x)$  defined by

$$f(x) = \begin{cases} 2 & \text{if } -\pi \leq x < 0 \\ -1 & \text{if } 0 \leq x < \pi \end{cases}$$

and having period  $2\pi$ . Does the Fourier series converge at  $x = 0$ ? If yes, what is the limit?

**7.3.** Let  $f$  be a real-valued  $2\pi$ -periodic function with Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

(a) Show that  $c_n = \overline{c_{-n}}$ .

Find (complex) Fourier coefficients for the following functions:

- (b)  $f(x - x_0)$  for some fixed  $x_0$ ;
- (c)  $f(-x)$ ;
- (d)  $f'(3x)$ .

**7.4.** (★) Compute Fourier series of function  $f(x) = x(\pi - x)$  on  $[0, \pi]$ . Use the Fourier series to compute the sum of the series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$$