Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 8

Due Date: 14:00 Wednesday, April 22.

<u>Explain your answers!</u> Problems marked ($\star$) are bonus ones.

8.1. Find the Fourier transform of the function $f(x) = \dfrac{1}{2\pi} e^{-x^2/2}$ (use the fact that $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$).

8.2. Let $\delta(x)$ be the delta function. Compute

(a) $\displaystyle\int_{-2}^{2} \delta(x)(2 - x^2 + e^x)\, dx$; (b) $\displaystyle\int_{-2}^{2} \delta(x+1)(1 + x^3 - \cos^3(\pi x))\, dx$;

(c) $\displaystyle\int_{-2}^{2} \delta(x-3)e^{-x^2} \sin x\, dx$; ($\star$) $\displaystyle\int_{-2\pi}^{2\pi} \delta(x^2 - \pi^2) \cos x\, dx$.

8.3. ($\star$) Consider the following equation

$$\frac{d^2u}{dx^2} - u(x) = f(x)$$

with respect to u. Show that the solution $u(x)$ can be written as

$$u(x) = \frac{-1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i\xi x}\, \hat{f}(\xi)}{1 + \xi^2}\, d\xi$$

where $\hat{f}(\xi)$ is the Fourier transform of $f(x)$.

8.4. Let $f * g$ be a convolution of two functions. Show that

(a) $f * g = g * f$;

(b) $(f * g) * h = f * (g * h)$;

(c) $f * (g + h) = f * g + f * h$.

(d) Is it true that $(f * g)h = f * (gh)$?

8.5. Compute Laplace transform of the following functions:

(a) $f(x) = x^n,\ \ n \in \mathbb{Z}$;

(b) $f(x) = \sin(\alpha x),\ \ \alpha \in \mathbb{R}$;

(c) $f(x) = \cos(\alpha x),\ \ \alpha \in \mathbb{R}$;

(d) $\delta(2x - x_0)$.