Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 9

Due Date: 14:00 Wednesday, April 29.

Explain your answers! Problems marked ($\star$) are bonus ones.

9.1. Determine whether the following relationships are valid.

(a) $\overline{(\overline{X} \cup Y)} = X \cap \overline{Y}$

(b) $(X \cup Y) \cap Z = (X \cup Z) \cap Y$

(c) $X \cup \overline{(Y \cap Z)} = (X \cup \overline{Y}) \cup \overline{Z}$

9.2. ($\star$) Given that events X, Y, and Z satisfy

$$(X \cap Y) \cup (Z \cap X) \cup \overline{(\overline{X} \cup \overline{Y})} = \overline{(Z \cup \overline{Y})} \cup \left[\left(\overline{(\overline{Z} \cup \overline{X})} \cup (\overline{X} \cap Z)\right) \cap Y\right],$$

prove that $X \supset Y$ and that $Z \cap X \subset Y$.

9.3. There are 9 different cups, 9 similar glasses, 4 different tea bags, and 4 similar tea spoons. How many choices do you have to distribute

(a) spoons amongst glasses;

(b) spoons amongst cups;

(c) tea bags amongst cups;

(d) tea bags amongst glasses?

(there are no restrictions on the number of items inside cups or glasses).

9.4. Compute the number of 7-digit phone numbers satisfying the following property:

(a) there are no "5" and "7" in the number;

(b) there are no two equal digits in a row;

(c) ($\star$) there is no "7" after "8";

(d) ($\star$) each digit is not less than the preceding one.

9.5. Compute the number of different 6-sided dice (with digits from 1 to 6 on their sides).

9.6. In a card game each of four players is dealt 13 cards from a full pack of 52. What is the probability that

(a) Player A gets two aces, players B and C each one ace;

(b) some player gets (exactly) two aces?

9.7. Let A and B be two mutually exclusive events. Suppose $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Compute the probabilities $P(A \,|\, B)$, $P(B \,|\, A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A \backslash B)$, $P(B \backslash A)$, $P(\overline{A} \,|\, B)$, $P(B \,|\, \overline{A})$.

9.8. *The Monty Hall Problem*

Suppose you are on a game show, and you are given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Explain the answer using the notion of conditional probability.