Jacobs University
School of Engineering and Science

Spring Term 2010

# ESM 2B, Homework 1

**Due Date:** 14:00 Thursday, 18 February 2010.

Explain your answers! Problems marked ($\star$) are bonus ones.

**1.1.** Are the following sets of vectors linearly independent?

(a) $\{(1,-1,0),\ (-2,0,2),\ (0,1,-1)\} \subset \mathbb{R}^3$;

(b) $\{(1,1,0),\ (1,0,1),\ (0,1,1)\} \subset \mathbb{R}^3$;

(c) $\{x^3+x^2,\ x^2+3x+1,\ x^3-2x^2+1\} \subset \mathbb{R}[x]$, where $\mathbb{R}[x]$ is the space of polynomials with $\mathbb{R}$-coefficients.

**1.2.** What is the dimension of $\text{span}\{(2,3,2),\ (3,2,1),\ (-1,6,3)\} \subset \mathbb{R}^3$?

**1.3.** Find all values of $a$ and $b$, such that $\{(a,1),(b,b)\}$ is a linearly dependent subset of $\mathbb{C}^2$.

**1.4.** Show that for any vector space $V$ and any $v \in V$ the product $1 \cdot v$ is equal to $v$.

**1.5.** Which of the following sets are $\mathbb{R}$-vector spaces?

(a) Polynomials with real coefficients of degree at least $n$;

(b) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0) = 0$;

(c) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0) = 1$;

(d) bounded functions on $\mathbb{R}$;

(e) arithmetic progressions with real entries;

(f) geometric progressions with real entries;

($\star$) the set of functions $\{f(x) = a\cos(x+c) \mid a, c \in \mathbb{R}\}$.

**1.6.** Which of the following maps $V \times V \to \mathbb{F}$ are inner products?

(a) $\mathbb{F} = \mathbb{C}$, $V = \mathbb{C}^2$, $\langle w|z\rangle = \overline{w_1}z_2 - \overline{w_2}z_1$;

(b) $\mathbb{F} = \mathbb{R}$, $V = \mathbb{R}^3$, $\langle x|y\rangle = x_1y_2 - x_2y_1 + 3x_3y_3$.

**1.7.** Which of the following maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ are linear operators?

(a) $f(x,y) = (xy, x-y)$;

(b) $g(x,y) = (x-y, 2x)$.

**1.8.** Which of the following maps are linear transformations?

(a) $\mathcal{A} : \mathbb{R}^4 \to \mathbb{R}^3$, $\mathcal{A}(x,y,z,t) = (x+2y, y+2z, z+2t)$;

($\star$) $\mathcal{A} : \mathbb{R}[x] \to \mathbb{R}[x]$, $(\mathcal{A}p)(x) = p(\alpha x^2 + \beta)$, where $\alpha, \beta \in \mathbb{R}$ are fixed numbers.

**1.9.** Show that for every linear map $\mathcal{A} : V \to W$ the kernel $\ker\mathcal{A}$ and the image $\operatorname{im}\mathcal{A}$ are linear subspaces of $V$ and $W$ respectively.