ESM 2B, Homework 10

Due Date: 14:00 Thursday, May 6.

Explain your answers! Problems marked ($\star$) are bonus ones.

10.1. Compute the number of 7-digit phone numbers satisfying the following property:

(a) there are no "6" and "7" in the number;

(b) there are two equal digits in a row;

(c) ($\star$) there is no "7" after "5";

(d) ($\star$) each digit is not greater than the preceding one.

10.2. In a card game each of four players is dealt 13 cards from a full pack of 52. What is the probability that

(a) Player A gets two aces, players B and C each one ace;

(b) some player gets (exactly) two aces?

10.3. Let A and B be two mutually exclusive events. Suppose $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Compute the probabilities $P(A \,|\, B)$, $P(B \,|\, A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A \setminus B)$, $P(B \setminus A)$, $P(\overline{A} \,|\, B)$, $P(B \,|\, \overline{A})$.

10.4. In how many ways can 9 people be placed around a table if there are three who insist on sitting together?

10.5. Prove the following identities:

(a) $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$;

(b) $\sum\limits_{k=0}^{n} \binom{n}{k} = 2^n$;

(c)($\star$) $\sum\limits_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$.

10.6. A boy is selected at random from among the children belonging to families with n children. What is the probability that the boy has $k - 1$ brothers?

10.7. Suppose we have a box with 10 balls, 6 of which are black and 4 are white. Find the probability of drawing 3 white balls from the box if:

(a) the first ball is returned into the box before the second ball is drawn;

(b) the first ball is put aside after being drawn.

10.8. *The Monty Hall Problem*

Suppose you are on a game show, and you are given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Explain the answer using the notion of conditional probability.