

ESM 2B, Homework 7

Due Date: 14:00 Thursday, April 15.

Explain your answers! Problems marked (★) are bonus ones.

7.1. Write down Fourier series for the following functions from $L^2([-\pi, \pi])$.

- (a) $f(x) = |x|$;
- (b) $f(x) = \sin^2(2x)$;
- (c) $f(x) = \cos^2(x)$;
- (d) $f(x) = \sin^2(x) - x$.

7.2. Write down Fourier series for the following functions from $L^2([0, \pi])$.

- (a) $f(x) = x$;
- (a) $f(x) = x^2$.

7.3. Find the Fourier series of the function $f(x)$ defined by

$$f(x) = \begin{cases} 1 & \text{if } -\pi \leq x < 0 \\ -2 & \text{if } 0 \leq x < \pi \end{cases}$$

and having period 2π . Does the Fourier series converge at $x = 0$? If yes, what is the limit?

7.4. Let f be a real-valued 2π -periodic function with Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

(a) Show that $c_n = \overline{c_{-n}}$.

Find (complex) Fourier coefficients for the following functions:

- (b) $f(x - x_0)$ for some fixed x_0 ;
- (c) $f(-x)$;
- (d) $f'(2x)$.

7.5. (★) Compute Fourier series of function $f(x) = x(\pi - x)$ on $[0, \pi]$. Use the Fourier series to compute the sum of the series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$$