Jacobs University
School of Engineering and Science

Spring Term 2010

ESM 2B, Homework 9

Due Date: 14:00 Thursday, April 29.

Explain your answers! Problems marked ($\star$) are bonus ones.

9.1. Compute Laplace transform of the following functions:

(a) $f(x) = x^n,\ \ n \in \mathbb{Z}$;

(b) $f(x) = \sin(\alpha x),\ \ \alpha \in \mathbb{R}$;

(c) $f(x) = \cos(\alpha x),\ \ \alpha \in \mathbb{R}$;

(d) $\delta(3x - x_0)$.

9.2. Show that

(a) $X \cup Y = X \iff X \cap Y = Y \iff Y \subset X$;

(b) $Y \subset X \iff \overline{X} \subset \overline{Y}$.

9.3. Determine whether the following relationships are valid.

(a) $\overline{(\overline{X} \cup Y)} = X \cap \overline{Y}$

(b) $(X \cup Y) \cap Z = (X \cup Z) \cap Y$

(c) $X \cup \overline{(Y \cap Z)} = (X \cup \overline{Y}) \cup \overline{Z}$

9.4. ($\star$) Given that events X, Y, and Z satisfy

$$(X \cap Y) \cup (Z \cap X) \cup \overline{(\overline{X} \cup \overline{Y})} = \overline{(Z \cup \overline{Y})} \cup \left[\left(\overline{(\overline{Z} \cup \overline{X})} \cup (\overline{X} \cap Z)\right) \cap Y\right],$$

prove that $X \supset Y$ and that $Z \cap X \subset Y$.

9.5. Compute the number of different 6-sided dice (with digits from 1 to 6 on their sides).

9.6. There are 7 different cups, 7 similar glasses, 3 different tea bags, and 3 similar tea spoons. How many choices do you have to distribute

(a) spoons amongst glasses;

(b) spoons amongst cups;

(c) tea bags amongst cups;

(d) tea bags amongst glasses?

(there are no restrictions on the number of items inside cups or glasses).