Jacobs University
School of Engineering and Science

Spring Term 2011

ESM 2B, Homework 1

Due Date: 14:00 Wednesday, 16 February 2011.

Explain your answers! Problems marked ($\star$) are bonus ones.

1.1. Are the following sets of vectors linearly independent?

(a) $\{(1, -1, 0),\ (-2, 0, 2),\ (0, 3, -3)\} \subset \mathbb{R}^3$;

(b) $\{(1, 1, 0),\ (2, 0, 2),\ (0, 1, 1)\} \subset \mathbb{R}^3$;

(c) $\{x^3 + x^2,\ x^2 + 3x + 1,\ x^3 - 5x^2 + 1\} \subset \mathbb{R}[x]$, where $\mathbb{R}[x]$ is the space of polynomials with $\mathbb{R}$-coefficients.

1.2. Is it possible that vectors $v_1, \ldots, v_5 \in \mathbb{R}^5$ are linearly dependent, but any 4 of them are linearly independent?

1.3. What is the dimension of $\operatorname{span}\{(2, 3, 2),\ (6, 4, 2),\ (-1, 6, 3)\} \subset \mathbb{R}^3$?

1.4. Find all values of a and b, such that $\{(1, a), (b, b)\}$ is a linearly dependent subset of $\mathbb{C}^2$.

1.5. Show that for any vector space V and any $v \in V$ the product $1 \cdot v$ is equal to v.

1.6. Which of the following sets are $\mathbb{R}$-vector spaces?

(a) Polynomials with real coefficients of degree at least n;

(b) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0) = 0$;

(c) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0) = 1$;

(d) bounded functions on $\mathbb{R}$;

(e) arithmetic progressions with real entries;

(f) geometric progressions with real entries;

($\star$) the set of functions $\{f(x) = a\sin(x + c) \,|\, a, c \in \mathbb{R}\}$.

1.7. Which of the following maps $V \times V \to \mathbb{F}$ are inner products?

(a) $\mathbb{F} = \mathbb{C}$, $V = \mathbb{C}^2$, $\langle w|z\rangle = \overline{w_1} z_2$;

(b) $\mathbb{F} = \mathbb{C}$, $V = \mathbb{C}^2$, $\langle w|z\rangle = \overline{w_1} z_2 - \overline{w_2} z_1$;

(c) $\mathbb{F} = \mathbb{R}$, $V = \mathbb{R}^3$, $\langle x|y\rangle = x_1 y_2 - x_2 y_1 + 3x_3 y_3$;

(d) $\mathbb{F} = \mathbb{R}$, $V = \mathbb{R}^3$, $\langle x|y\rangle = x_1 y_2 + x_2 y_1 + 3x_3 y_3$.

1.8. ($\star$) Find dimensions of linear spaces from Exercise 1.6 (or show that they do not have finite bases).