## ESM 2B, Homework 10

**Due Date:** 14:00 Wednesday, 4 May 2011.

Explain your answers! Problems marked $(\star)$ are bonus ones.

**10.1.** Show that

(a) $X \cup Y = X \iff X \cap Y = Y \iff Y \subset X$;

(b) $Y \subset X \iff \overline{X} \subset \overline{Y}$.

**10.2.** Determine whether the following relationships are valid.

(a) $\overline{(\overline{X} \cup Y)} = X \cap \overline{Y}$

(b) $(X \cup Y) \cap Z = (X \cup Z) \cap Y$

(c) $X \cup \overline{(Y \cap Z)} = (X \cup \overline{Y}) \cup \overline{Z}$

(d) $\overline{X \setminus Y} = Y \cup \overline{X \cup Y)}$

**10.3.** $(\star)$ Given that events $X, Y$, and $Z$ satisfy

$$(X \cap Y) \cup (Y \cap Z) \cup \overline{(\overline{X} \cup \overline{Y})} = \overline{(Z \cup \overline{X})} \cup \left[ \left( (\overline{Z} \cup \overline{Y}) \cup (\overline{Y} \cap Z) \right) \cap X \right],$$

prove that $X \subset Y$ and that $Z \cap Y \subset X$.

**10.4.** Compute the number of different 6-sided dice (with digits from 1 to 6 on their sides).

**10.5.** A class consists of 79 students, 40 of them speak German, 42 speak Spanish, 25 speak German and French, 22 speak German and Spanish, 15 speak all the three languages, and 20 speak French but not Spanish. How many students do not speak any of the three languages?