Jacobs University
School of Engineering and Science

Spring Term 2011

ESM 2B, Homework 11

Due Date: 14:00 Wednesday, May 11.

Explain your answers! Problems marked ($\star$) are bonus ones.

11.1. Compute the number of 8-digit phone numbers satisfying the following property:

(a) there are no "5" and "7" in the number;

(b) there are two equal digits in a row;

(c) ($\star$) there is no "6" after "8";

(d) ($\star$) each digit is not smaller than the preceding one.

11.2. In a card game each of four players is dealt 13 cards from a full pack of 52. What is the probability that

(a) Player A gets two aces, players B and C each one ace;

(b) some player gets (exactly) two aces?

11.3. Let A and B be two mutually exclusive events. Suppose $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$. Compute the probabilities $P(A \,|\, B)$, $P(B \,|\, A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A \setminus B)$, $P(B \setminus A)$, $P(\overline{A} \,|\, B)$, $P(B \,|\, \overline{A})$.

11.4. In how many ways can 8 people be placed around a table if there are four who insist on sitting together?

11.5. Prove the following identities:

(a) $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$;

(b) $\sum\limits_{k=0}^{n} \binom{n}{k} = 2^n$;

(c)($\star$) $\sum\limits_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$;

(d)($\star$) $\binom{n}{k} = \sum\limits_{m=0}^{k} \binom{k}{m}\binom{n-k}{k-m}$.

11.6. A boy is selected at random from among the children belonging to families with n children. What is the probability that the boy has $k-1$ brothers?

11.7. Suppose we have a box with 10 balls, 3 of which are black and 7 are white. Find the probability of drawing 3 white balls from the box if:

(a) the first ball is returned into the box before the second ball is drawn;

(b) the first ball is put aside after being drawn.

11.8. Suppose one in 101 people has a certain disease. There is a test for this disease that is completely reliable in detecting the disease but gives a false positive result on 1% of the noninfected population. Given that somebody is tested positively, what is the probability that this person indeed has the disease?
Hint: Consider events A ={Test is positive} and B ={Person is infected}, and then use Bayes' rule.