Jacobs University
School of Engineering and Science

Spring Term 2011

ESM 2B, Homework 12

Due Date: Tuesday, May 17.

<u>Explain your answers!</u> Problems marked ($\star$) are bonus ones.

12.1. Let $X_1,\ X_2,\ \ldots$ be independent discrete random variables taking values in $\{-\frac{1}{2}, \frac{1}{2}\}$ with probability function

$$p(-\tfrac{1}{2}) = q\,, \qquad p(\tfrac{1}{2}) = (1-q)\,,$$

and $p(x) = 0$ for all other values of x.

(a) Find the expectation for the random variable $Z_N = \frac{X_1+\cdots+X_N}{N}$ as $N \to \infty$.

(b) Let $Y_n = 2^{-n}X_n$. Find the expectation for the random variable $Z_N = Y_1 + \cdots + Y_N$ as $N \to \infty$.

12.2. Assume Peter leaves his office every day at a random time, goes to the bus stop, from where every 7 minutes leaves a bus.

(a) Determine the expected waiting time.

(b) After four days, Peter observed that he never waited longer than 6 minutes. What is the probability of this event?

(c) What is the probability that Peter waited at least one time longer than 5 minutes in 3 days?

12.3. The moment generating function of a certain random variable X is $M_X(t) = \dfrac{c}{c-t}$, where $c > 0$ is a parameter. Determine the expectation and variance of X.

12.4. Suppose a random number generator produces uniformly distributed numbers X in the interval $[1, 3]$, and another random number generator produces uniformly distributed numbers Y in the interval $[2, 5]$. What is the probability that $X + Y \leq 5$?

12.5. ($\star$) Let X and Y be independent random variables that take non-negative real values and whose associated density functions are $f_X(t) = \lambda e^{-\lambda t}$, $f_Y(t) = \mu e^{-\mu t}$ for $t > 0$, and $f_X(t) = f_Y(t) = 0$ for $t < 0$. Compute the probability $P(a < X + Y < b)$ for real $a < b$.