

ESM 2B, Homework 4

Due Date: 14:00 Wednesday, 9 March 2011.

Explain your answers! Problems marked (\star) are bonus ones.

4.1. Are the following matrices invertible? If yes, compute the inverses.

$$(a) \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad (b) \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 4 & 3 & 5 \end{pmatrix} \quad (c) \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 1 \\ 4 & 0 & 2 \end{pmatrix} \quad (d) \begin{pmatrix} 2 & 1 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

4.2. Compute determinants of the following matrices:

$$(a) \begin{pmatrix} 0 & 1 & 2 \\ 1 & 3 & 1 \\ 3 & 2 & -1 \end{pmatrix} \quad (b) \begin{pmatrix} 2 & 0 & -2 \\ 3 & 2 & -3 \\ -2 & 2 & 2 \end{pmatrix} \quad (c) \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ 4 & 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{pmatrix}$$

$$(d) \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{pmatrix} \quad (e) \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (f) \begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

4.3. (a) Show that if A is an invertible matrix, then A^n is also invertible for any positive integer n .

(b) Is it true that if A^n is invertible for some positive integer n then A is invertible?

(c) Find matrices $A \in \text{Mat}_{n \times m}$ and $B \in \text{Mat}_{m \times n}$ such that $AB = I$, but $BA \neq I$.

4.4. (\star) Let $A \in \text{Mat}_n$, $\det A = 0$. Show that there exist $B, C \in \text{Mat}_n$, such that

$$AB = CA = 0$$

4.5. (\star) Let $A, B \in \text{Mat}_n$, and let $AB + I$ be invertible. Show that $BA + I$ is also invertible.