

ESM 2B, Homework 6

Due Date: 14:00 Wednesday, 23 March 2011.

Explain your answers! Problems marked (★) are bonus ones.

6.1. Use the Gram-Schmidt procedure to

(a) construct an orthonormal set of vectors from

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad v_3 = \begin{pmatrix} 3 \\ 3 \\ 1 \\ -3 \end{pmatrix} \quad v_4 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

(b) find an orthonormal basis for the subspace of \mathbb{R}^3 spanned by

$$v_1 = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

6.2. Let $A = (a_{ij})$ be an orthogonal (or unitary) matrix. Show that $|a_{ij}| \leq 1$ for any i, j .

6.3. Find the Jordan normal form and the associated basis of the following matrices:

$$(a) \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \quad (b) \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix} \quad (c) \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad (\star) \begin{pmatrix} 3 & -1 & 1 & 7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix}$$

6.4. Compute

$$(a) \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}^{15} \quad (b) \begin{pmatrix} -1 & -1 \\ 4 & 3 \end{pmatrix}^{20}$$

6.5. (★) Find eigenvalues, eigenvectors, generalized eigenspaces and Jordan normal form of the differentiation operator on the space of polynomials of degree at most n .