Jacobs University
School of Engineering and Science

Spring Term 2011

ESM 2B, Homework 9

Due Date: 14:00 Wednesday, 27 April 2011.

Explain your answers! Problems marked ($\star$) are bonus ones.

9.1. Find the Fourier transform of the following functions:

(a) $f(x) = e^{-\alpha x^2}$ (use the fact that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$);

(b) $f(x) = xe^{-|x|}$; (c)($\star$) $f(x) = \sin x e^{-|x|}$.

9.2. Assuming Fourier transform of a function $f(x)$ to be $\hat{f}(y)$, compute the Fourier transform of
(a) $g(x) = f(x-a)$, $a \in \mathbb{R}$; (b) $g(x) = f(x/a)$, $a > 0$.

9.3. Consider the following equation

$$\frac{d^2u}{dx^2} - u(x) = f(x)$$

with respect to u. Show that the solution $u(x)$ can be written as

$$u(x) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{iyx}\,\hat{f}(y)}{1+y^2}\,dy$$

where $\hat{f}(y)$ is the Fourier transform of $f(x)$.

9.4. Let $f * g$ be a convolution of two functions. Show that
(a) $f * g = g * f$; (b) $(f * g) * h = f * (g * h)$; (c) $f * (g + h) = f * g + f * h$.
(d) Is it true that $(f * g)h = f * (gh)$?

9.5. Let $\delta(x)$ be the δ-function. Compute

(a) $\int_{-3}^{2} \delta(x)(2 - 2x^2 + e^x)\,dx$; (b) $\int_{-3}^{2} \delta(x+1)(1 + 2x^3 - \cos^3(\pi x))\,dx$;

(c) $\int_{-3}^{2} \delta(x-3)e^{-2x^2}\cos x\,dx$; (d)($\star$) $\int_{-2\pi}^{2\pi} \delta(x^2-\pi^2)\cos x\,dx$; (e)($\star$) $\int_{-2\pi}^{2\pi} \delta(4x^2-\pi^2)\sin x\,dx$.

9.6. Compute Laplace transform of the following functions:
(a) $f(x) = 2$; (b) $f(x) = x^n$; (c)($\star$) $f(x) = \sin \alpha x$; (d)($\star$) $f(x) = x\cos \alpha x$