

Linear Algebra II, Homework 1

Due Date: Wednesday, February 16, in class.

Problems marked (★) are bonus ones.

1.1. Find the Jordan normal form and the associated basis of the following matrices:

$$(a) \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \quad (b) \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix} \quad (c) \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad (d) \begin{pmatrix} 3 & -1 & 1 & 7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix}$$

1.2. Compute

$$(a) \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}^{15} \quad (b) \begin{pmatrix} -1 & -1 \\ 4 & 3 \end{pmatrix}^{20} \quad (c) \begin{pmatrix} 1 & -3 & 4 \\ 4 & -7 & 8 \\ 6 & -7 & 7 \end{pmatrix}^{30} \quad (d) \exp \begin{pmatrix} 1 & -4 \\ -4 & 8 \end{pmatrix}$$

1.3. Find eigenvalues, eigenvectors, generalized eigenspaces and Jordan normal form of the following operators on the space of real polynomials of degree at most n :

- (a) $p(x) \rightarrow p'(x)$;
- (b) $p(x) \rightarrow p^{(k)}(x)$.

1.4. (★) Show that characteristic polynomials of matrices AB and BA coincide.