Jacobs University
School of Engineering and Science

Spring Term 2011

Linear Algebra II, Homework 10

Due Date: Wednesday, April 27, in class.

Problems marked ($\star$) are bonus ones.

10.1. Show that ellipse, hyperbola, and parabola on the plane are projectively equivalent.

10.2. In 3-dimensional space four planes pass through a line l, and a line m intersects all the four planes (but not the line l). Show that the cross-ratio of the intersection points of m with the planes does not depend on the choice of m.

The points $x_1, \ldots, x_k$ are *collinear* if they belong to one line.

10.3. (a) How many distinct values does the cross-ratio of four points on a line take when the order of the points is changed?

(b) Given five distinct collinear points x_1, x_2, x_3, x_4, x_5, show that

$$[x_1, x_2, x_3, x_4][x_1, x_2, x_4, x_5][x_1, x_2, x_5, x_3] = 1$$

10.4. Let $\rho = [x_1, x_2, x_3, x_4]$ be cross-ratio of four points. Show that the value

$$\frac{(\rho^2 - \rho + 1)^3}{\rho^2(\rho - 1)^2}$$

is invariant under permutations of points x_1, x_2, x_3, x_4.

10.5. ($\star$) [Pascal's Theorem] The points $x_1, \ldots, x_6$ lie on ellips. Let y_1, y_2, y_3 be the intersection points of lines x_1x_2 and x_4x_5, x_1x_6 and x_3x_4, x_2x_3 and x_5x_6, respectively. Then the points y_1, y_2, y_3 are collinear.