Jacobs University
School of Engineering and Science

Spring Term 2011

Linear Algebra II, Homework 11

Due Date: Wednesday, May 4, in class.

Problems marked ($\star$) are bonus ones.

11.1. Let L be an n-dimensional vector space. Show that $PGL(L)$ acts transitively on

(a) the set of points of $\mathbb{P}(L)$;

(b) the set of pairs (M_1, M_2) of subspaces of $\mathbb{P}(L)$, such that $\dim M_1, \dim M_2$ and $\dim M_1 \cap M_2$ are fixed;

(c)($\star$) the set of *flags*, where *flag* is a collection of subspaces

$$M_0 \subsetneq M_2 \subsetneq \cdots \subsetneq M_{n-2} \subsetneq \mathbb{P}(L), \quad \dim M_i = i$$

11.2. Let $L_1, L_2 \subset L$, $M_1, M_2 \subset M$. Show that

$$(L_1 \otimes M_1) \cap (L_2 \otimes M_2) = (L_1 \otimes L_2) \cap (M_1 \otimes M_2)$$

11.3. Show that the natural map $t : L_1 \times L_2 \to L_1 \otimes L_2$ is bilinear.

11.4. Show that for any factorizable $x \in L_1 \otimes L_2$ there is a unique representation

$$x = x_1 \otimes x_2 \qquad x_1 \in L_1, x_2 \in L_2$$

up to transformation $x_1 \to \lambda x_1$, $x_2 \to \lambda^{-1} x_2$.

11.5. ($\star$) Let $A(x, y)$ be a bilinear function on a Euclidean space L. Suppose also that $A(x, y) = 0$ as soon as $g(x, y) = 0$. Show that A is proportional to g.