Jacobs University
School of Engineering and Science

Spring Term 2011

Linear Algebra II, Homework 5

Due Date: Wednesday, March 16, in class.

Problems marked $(\star)$ are bonus ones.

5.1. Use Gram-Schmidt procedure to construct an orthonormal basis of the space of real polynomials of degree at most 4 with inner product $g(p_1, p_2) = \int_0^1 p_1 p_2$ starting with basis $\{1, x, x^2, x^3, x^4\}$.

5.2. Let L be non-degenerate finite-dimensional orthogonal space, $L_0 \subset L$ is non-degenerate subspace. Show that for any orthogonal basis $\{e_1, \ldots, e_k\}$ of L_0 there exists an orthogonal basis $\{e_1, \ldots, e_k, e_{k+1}, \ldots, e_n\}$ of L containing $\{e_1, \ldots, e_k\}$.

5.3. Find a basis of $\mathbb{R}^3$ in which the quadratic form $q(x, y, z) = xy + 2yz + zx$ is diagonal with coefficients ± 1 or 0.

5.4. Find the signature of the quadratic form defined by the matrix

$$\begin{pmatrix} 2 & 0 & 3 & -1 \\ 0 & 1 & 2 & 2 \\ 3 & 2 & -2 & 0 \\ -1 & 2 & 0 & 2 \end{pmatrix}$$

5.5. $(\star)$ Find the signature of the quadratic form on $\mathbb{R}^n$ defined by

$$q(x) = \sum_{i<j} (x_i - x_j)^2$$