Jacobs University
School of Engineering and Science

Spring Term 2011

Linear Algebra II, Homework 6

Due Date: Wednesday, March 23, in class.

Problems marked ($\star$) are bonus ones.

6.1. Let $f : L \to L$ be an endomorphism of unitary space L. Prove that

(a) if $(f(x), x) = 0$ for all $x \in L$ then $f = 0$;

(b) if $(f(x), x) \in \mathbb{R}$ for all $x \in L$ then f is Hermitian.

6.2. (a) Every square complex matrix A can be represented in a unique way as a sum $A = B + iC$, where B and C are Hermitian matrices.

(b) Every square complex matrix A can be represented in a unique way as a sum of Hermitian and skew-Hermitian matrices.

6.3. (a) Show that matrices of type

$$\begin{pmatrix} x+iy & z+iw \\ -z+iw & x-iy \end{pmatrix}, \quad x, y, z, w \in \mathbb{R}, \quad x^2+y^2+z^2+w^2 = 1$$

form special unitary group $SU(2)$.

(b) Show that matrices of type

$$\begin{pmatrix} iy & z+iw \\ -z+iw & -iy \end{pmatrix}, \quad y, z, w \in \mathbb{R}$$

with quadratic form $q(A) = \det A$ compose a 3-dimensional Euclidean space E.

(c) Show that the map $f : SU(2) \to SO(3)$ defined by

$$f(U)(A) = UAU^{-1}$$

for $U \in SU(2)$, $A \in E$ is a homomorphism of groups.

(d)($\star$) Show that f is surjective, and $\ker f = \pm I$.