Jacobs University
School of Engineering and Science
Spring Term 2011

Linear Algebra II, Homework 9

Due Date: Wednesday, April 13, in class.

Problems marked $(\star)$ are bonus ones.

9.1. Let A be a skew-symmetric real matrix.

(a) Show that A^2 is symmetric, and the orthogonal inner product

$$(x, y) = x^t A^2 y$$

is non-positive definite.

(b) Show that all non-zero eigenvalues of A are purely imaginary.

(c)$(\star)$ Is it true that if A is non-degenerate, then A^{-1} is also skew-symmetric?

9.2. Show that for skew-symmetric 4×4 matrix $A = (a_{ij})$ its Pffafian can be written as

$$\text{Pf } A = -a_{12}a_{34} + a_{13}a_{24} - a_{14}a_{23}$$

9.3. Let L be a linear space, $M_1, M_2 \subset L$ are linear subspaces. For any linear space M denote by $P(M)$ the projectivization of M. Show that

(a) $P(M_1) \cap P(M_2) = P(M_1 \cap M_2)$;

(b) $P(M_1 + M_2)$ coincides with the projective span $\overline{P(M_1) \cup P(M_2)}$ of $P(M_1) \cup P(M_2)$.

9.4. Let P_1 be a projective line, and $P_2 \notin P_1$ be a point in projective space $\mathbb{F}\mathrm{P}^4$. Find $\overline{P_1 \cup P_2}$.

9.5. (a) Show that the system of linear equations

$$\sum_{j=0}^{n} a_{ij} x_j = 0, \qquad 1 \le i \le m$$

defines a projective subspace in $\mathbb{F}\mathrm{P}^n$.

(b)$(\star)$ Show that any projective subspace in $\mathbb{F}\mathrm{P}^n$ can be defined by a system of linear homogeneous equations.