Durham University
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Epiphany 2012

Riemannian Geometry IV, Homework 4 (Week 15)

Due date for starred problems: **Wednesday, March 14**.

4.1. Compute (sectional) curvature of 2-dimensional sphere of radius r in $\mathbb{R}^3$.

4.2. ($\star$) Let (M, g) be a Riemannian manifold. Show that M is of constant sectional curvature K_0 if and only if

$$\langle R(v_1, v_2)v_3, v_4\rangle = K_0(\langle v_1, v_4\rangle\langle v_2, v_3\rangle - \langle v_1, v_3\rangle\langle v_2, v_4\rangle)$$

for any $p \in M$ and $v_1, v_2, v_3, v_4 \in T_pM$.

4.3. (a) Show that sectional curvature $K(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_3})$ and $K(\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3})$ of 3-dimensional real hyperbolic space $\mathbb{H}^3$ (in upper half-space model) is equal to -1.

(b) Use (a) and linearity of Riemann curvature tensor to show that 3-dimensional real hyperbolic space $\mathbb{H}^3$ has constant sectional curvature.