Durham University
Pavel Tumarkin Epiphany 2019

Geometry III/IV, Problems Class 1

Wednesday, January 30

Möbius transformations, inversion

P1.1. Find the type of the Möbius transformation $f(z) = \frac{1}{z}$.

P1.2. Let f, g be reflections or inversions. Show that $g \circ f = f \circ g$ if and only if Fix_f is orthogonal to Fix_g.

P1.3. Let $\gamma_1, \dots, \gamma_5$ be circle all passing through the same points $A, B \in \mathbb{R}^2$. Show that there exists a circle $\mathcal{C}$ orthogonal to all circles γ_i.

P1.4. Prove Ptolemy's Theorem: for a cyclic quadrilateral $ABCD$ holds
$|AB| \cdot |CD| + |BC| \cdot |DA| = |AC| \cdot |BD|$.

(three proofs: (a) by using cross-ratios, as in HW; (b) by using inversion; (c) "proof without words").