Durham University
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Epiphany 2019

Geometry III/IV, Problems Class 2

Wednesday, February 14

Poincaré disc model of hyperbolic geometry

P2.1. Show that any two divergent lines have a unique common perpendicular
(i.e. if $l_1, l_2 \subset \mathbb{H}^2$ are divergent then there exists a unique line l' such that $l' \perp l_1$ and $l' \perp l_2$).

Definition. A hyperbolic polygon with all vertices on the absolute is called an *ideal* polygon.
Remark: Ideal polygons have zero angles.

P2.2. (a) Show that up to applying an isometry, there exists a unique hyperbolic ideal triangle.

(b) Show that hyperbolic ideal quadrilaterals modulo isometries form a 1-parameter family.

(c) How many hyperbolic ideal n-gons are there?

P2.3. Let $\gamma_0, \dots \gamma_3$ be circles in $\mathbb{E}^2$ such that γ_i is tangent to γ_{i+1} for all i (where i is considered modulo 4). Show that all the common points $\gamma_i \cap \gamma_{i+1}$ of the circles lie on on one circle or line.