Durham University
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Epiphany 2019

Geometry III/IV, Problems Class 4

Wednesday, March 14

Computations in projective models

P4.0. Homework problem 16.2 on the altitudes of a hyperbolic triangle.

Use the Klein model to prove that in a right-angled triangle with right angle γ the following hold:

P4.1. $\sinh a = \sinh c \sin \alpha$

P4.2. (Pythagoras Theorem) $\cosh c = \cosh a \cosh b$

P4.3. $\tanh b = \tanh c \cos \alpha$

P4.4. Use the Klein model to find the radius of the circle inscribed in the ideal triangle.

P4.5. (Lambert quadrilateral)

Let $ABCD$ be a hyperbolic quadrilateral with three right angles A, B, C and $\angle D = \varphi$. Denote $BA = a$, $BC = b$. Show that $\sinh a \sinh b = \cos \varphi$.