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Epiphany 2019

Geometry III/IV, Hints: weeks 15–16

Elementary hyperbolic geometry

15.1. Compute in the upper half-plane (don't forget first to move the triangle to a convenient place).

15.2.
- Use the notation as in the figure below.
- First, show that
$$\sin^2\alpha = \frac{(2k\cos\varphi)^2}{(1-k^2)^2 + 4k^2\cos^2\varphi}.$$
- Square the required expressions, express $\tanh^2$ and $\sinh^2$ through $\cosh^2$ and use the distance formula to get the latter.

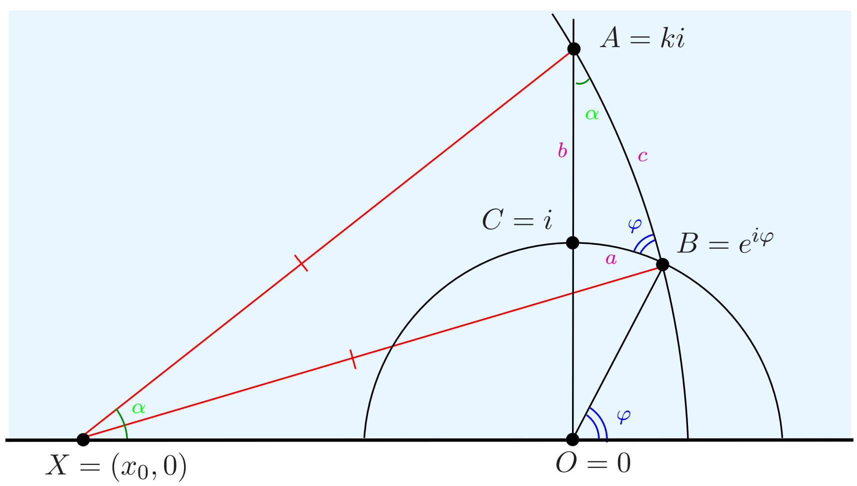

Figure 1: Notation for Problem 15.2

15.3. Use the identities on sinh and cosh.

15.5. Take one point on a given distance from the line and apply isometries to get more points on the same distance.

16.2. Place your triangle in the Klein model in such a way that all altitudes will be represented by the altitudes of a Euclidean triangle.

16.3. To compute, place the objects so that the required distance will be a length of a segment lying in the plane $x_2 = 0$, then everything is reduced to a 2-dimensional problem.

16.4. Use formulae listed in 16. 3.