Durham University
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Epiphany 2020

Riemannian Geometry IV, Problems class 4 (Week 20)

P4.1. Let $c : [0, a] \to M$ be a geodesic. Find explicitly geodesic variations $F(s, t)$ and $F^0(s, t)$ of c such that their variational vector fields are non-orthogonal Jacobi fields $tc'(t)$ and $c'(t)$ respectively.

P4.2. Let $\mathbb{H}^3 = \{(x, y, z) \in \mathbb{R}^3 \mid z > 0\}$ be the upper half-space model of the 3-dimensional hyperbolic space, where the metric is given by $(g_{ij}) = \frac{1}{z^2} I$. Given $a \in \mathbb{R}_{>0}$, show that the transformation $f_a : (x, y, z) \mapsto (ax, ay, az)$ is an isometry of $\mathbb{H}^3$.

P4.3. Show that the cone $z^2 = x^2 + y^2$ in $\mathbb{H}^3$ is isometric to Euclidean cylinder $x^2 + y^2 = 1$ in $\mathbb{R}^3$.

P4.4. Let G be a Lie group with bi-invariant metric. Show that it has non-negative sectional curvature.