

Topics in Combinatorics IV, Homework 15 (Week 15)

Due date for starred problems: **Monday, March 2, 12pm.**

- 15.1.** Let $\Gamma = \langle s_1, s_2, s_3 \mid s_i^2, (s_1 s_2)^3, (s_2 s_3)^3, (s_1 s_3)^3, s_3^7 \rangle$. Show that the subgroup generated by s_1 and s_2 is trivial, although the group $\Gamma' = \langle s_1, s_2 \mid \text{all relations not containing } s_3 \rangle$ is not.
- 15.2.** Let (G, S) be a Coxeter system, and let $T \subset S$. Define G_T to be the subgroup of G generated by elements of T (G_T is called a *standard parabolic subgroup* of G).
- (a) Let $w = s_1 \dots s_k$ be a word, all $s_i \in T$. Show that for any M -reduction $w \rightarrow w_0$ all words obtained during the procedure belong to G_T .
 - (b) Let $\Gamma = \langle T \mid s_i^2, (s_i s_j)^{m_{ij}} \rangle$. Define a homomorphism $\varphi : \Gamma \rightarrow G$ by $\varphi(s_i) = s_i$. Show that $\ker \varphi$ is trivial.
 - (c) Show that (G_T, T) is a Coxeter system.
- 15.3.** ($\star$) Let (G, S) be a Coxeter system, $s, t \in S$, and $m_{st} = \infty$ (i.e., there is no relation on st). Let w be a reduced word. Show that either $s \notin r(w)$ or $t \notin r(w)$.
- 15.4.** Let (G, S) be Coxeter system, $r \in R$ and $g \in G$. Show that if $r \in R(g)$ then $l(rg) < l(g)$.
- 15.5.** ($\star$) Let (G, S) be Coxeter system such that its Coxeter diagram contains a cycle. Find an element of infinite order in G .