Gauge theory and its application to geometry and low-dimensional topology

Regensburg, July 17–21, 2023

Abstracts for Lecture Series

Mike Miller Eismeier: Lecture series on transversality and gluing for (framed) instantons

Abstract for series: I will discuss the foundational aspects of instanton homology for rational homology 3-spheres. It is somewhere between necessary and convenient to study "framed" instanton moduli spaces and keep track of a leftover SO(3) symmetry. We will show that these framed moduli spaces have the right properties to be used in a definition of equivariant Floer homology, and how the framed picture relates to the more common unframed setting.

Lecture no. 1: Narrative overview

Lecture no. 2: Achieving transversality

Lecture no. 3: Boundary relations (gluing)

Lecture no. 4: Compactness of moduli spaces

Hokuto Konno: Lecture series on Seiberg-Witten theory for families

Abstract: Diffeomorphism groups of 4-manifolds are actively studied recently, and Seiberg-Witten theory for families of 4-manifolds is a powerful tool in this area. In this series of lectures, I aim to provide an overview of major techniques and applications of Seiberg-Witten theory for families. The first two lectures will focus on Seiberg-Witten-type invariants of families of 4-manifolds. The last two lectures will be about constraints on smooth families of 4-manifolds, which are families generalizations of work by Donaldson and Froyshov for negative-definite 4-manifolds.

Lecture no. 1: Families Seiberg-Witten invariant

Lecture no. 2: How to compute families Seiberg-Witten invariants

Lecture no. 3: Donaldson diagonalization for families

Lecture no. 4: Frøyshov inequality for families

Abstracts for the Research Talks

Mike Miller Eismeier (Columbia University, USA)

Obstructed gluing and applications

In the previous series, it was necessary to make certain index assumptions on the relevant cobordisms. Dropping these assumptions, one still obtains interesting moduli spaces by using obstructed gluing theory, but it's harder to extract topologically invariant information from these. I will discuss how to do so, and some applications of the technique.

Kim Anders Frøyshov (University of Oslo, Norway)

Mod 2 instanton homology: The additivity of q_2

I will discuss an integer invariant $q_2(Y)$ of oriented integral homology 3–spheres Y defined in terms of instanton cohomology with coefficients in $Z/2$. This invariant may be regarded as a mod 2 analogue of the h–invariant, which was defined with rational coefficients. Both invariants grew out of efforts to extend Donaldson's diagonalization theorem to 4–manifolds with boundary. The additivity of q_2 with respect to connected sums is a rather non-trivial fact, and I will explain some of the main ingredients of the proof.

Paolo Ghiggini (Université de Grenoble, France)

Bordered Floer bimodules from compact objects and genus two mutations

To every surface we associate an A-inffinity category whose objects are Heegaard multicurves, and which can be regarded as a cylindrical reformulation of the compact Fukaya category of the symmetric product. Moreover, to every cobordism between surfaces, decorated by a fixed handle decomposition, we associate an A-infinity functor between the categories of the boundary components. Finally we use results of Auroux to relate those functors to type AA bimodules. As an application, we prove that the total rank of Heegaard Floer homology (in the hat version) is preserved by genus two mutations. This is a work in progress in collaboration with Ina Petkova.

Marco Golla (Université de Nantes, France)

Signatures of aspherical 4-manifolds

A conjecture attributed to Singer stipulates that most L^2 Betti numbers of an aspherical manifold vanish. In dimension 4, this implies a conjecture of Gromov: the Euler characteristic of an aspherical 4-manifold bounds its signature. I will talk about a proof of Gromov's conjecture for geometrically decomposable 4-manifolds. This is joint work with Luca F. Di Cerbo.

Andras Juhasz (University of Oxford, UK)

The unknotting number, hard unknot diagrams, and Reinforcement Learning

We have developed a Reinforcement Learning agent based on the IMPALA architecture that often finds minimal unknotting trajectories for a knot diagram up to 200 crossings. We have used this to determine the unknotting number of 57k knots. We then took diagrams of connected sums of such knots with oppositely signed signatures, where the summands were overlaid. The agent has found unknotting trajectories involving several crossing changes that result in hyperbolic knots. Based on this, we have shown that, given knots K and K' that are not 2-bridge, there is a diagram of their connected sum and u(K) + u(K') unknotting crossings such that changing any one of them results in a prime knot. As a by-product, we have obtained a dataset of 2.6 million distinct hard unknot diagrams; most of them under 35 crossings. Assuming the additivity of the unknotting number, we can determine the unknotting number of 43 at most 12-crossing knots for which the unknotting number is unknown. This is joint work with Taylor Applebaum, Sam Blackwell, Alex Davies, Thomas Edlich, Marc Lackenby, Nenad Tomasev, and Daniel Zheng.

Hokuto Konno (University of Tokyo, Japan)

Exotic Dehn twists on 4-manifolds

A self-diffeomorphism of a smooth manifold is said to be exotic if it is topologically isotopic to the identity but smoothly not. We provide the first examples of exotic diffeomorphisms (in a relative sense) of contractible 4-manifolds, more generally of definite 4-manifolds. Such examples are given as Dehn twists along certain Seifert homology 3-spheres, which also give new examples of exotic diffeomorphisms that survive after one stabilization, and the smallest closed 4-manifold known to support an exotic diffeomorphism. The proof uses families Seiberg-Witten theory over $\mathbb{RP}^2$. This is joint work with Abhishek Mallick and Masaki Taniguchi.

Langte Ma (Stony Brook University, USA)

Homology $S^1 \times S^3$ and TQFT

In this talk, I will survey recent developments on the study of gauge theoretical invariants defined over closed smooth four manifolds whose integral homology is the same as that of $S^1 \times S^3$. The Seiberg–Witten type invariant defined by Mrowka–Ruberman–Salieve is proved to satisfy TQFT properties. Conjecturally, it is equivalent to the Yang–Mills type invariant introduced by Furuta–Ohta. The equivalence between these two invariants will lead to interesting consequences, including a conjecture of Akbulut.

Tomasz Mrowka (MIT Boston, USA)

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Katherine Raoux (University of Arkansas, USA)

A rational slice genus bound

We present a lower bound for the "rational slice genus" of a knot. The rational slice genus is a 4-D analogue of Calegari and Gordon's rational Seifert genus. And, our bound is a difference of Heegaard Floer tau invariants. To achieveit, we use relative adjunction inequalities applied to rational slice surfaces in a 3-manifold cross interval. Varying the boundary conditions of the surfaces gives us an infinite family of bounds and the minimum taken over all these is a universal lower bound.Along the way, we bound the tau invariants of braided satellites, compute the rational slice genus of Floer simple knots, and produce PL slice genus bounds for null-homologous knots. This is joint work with Matthew Hedden.

Masaki Taniguchi (Kyoto University, Japan)

Real Seiberg-Witten Floer Homotopy and Its Applications

In this talk, we present a development of a real version of the Seiberg-Witten Floer homotopy type for knots. This construction associates a $\mathbb{Z}/4\mathbb{Z}$-equivariant stable homotopy type of a space for a given knot. By utilizing this framework, we introduce several knot/link concordance invariants, namely δ_R and κ_R, defined as Froyshov-type invariants in relation to equivariant singular/K cohomology. We also examine various inequalities associated with these invariants and give their topological applications. This is joint work with Jin Miyazawa and Hokuto Konno.

Thomas Walpuski (Humboldt University Berlin)

The Gopakumar–Vafa finiteness conjecture

The purpose of this talk is to illustrate an application of the powerful machinery of geometric measure theory to a conjecture in Gromov–Witten theory arising from physics. Very roughly speaking, the Gromov–Witten invariants of a symplectic manifold (X, ω) equipped with a tamed almost complex structure J are obtained by counting pseudo-holomorphic maps from mildly singular Riemann surfaces into (X, J). It turns out that Gromov–Witten invariants are quite complicated (or "have a rich internal structure"). This is true especially for if (X, ω) is a symplectic Calabi–Yau 3–fold (that is: $dimX = 6$, $c_1(X, \omega) = 0$).

In 1998, using arguments from M–theory, Gopakumar and Vafa argued that there are integer BPS invariants of symplectic Calabi–Yau 3–folds. Unfortunately, they did not give a direct mathematical definition of their BPS invariants, but they predicted that they are related to the Gromov–Witten invariants by a transformation of the generating series. The Gopakumar–Vafa conjecture asserts that if one defines the BPS invariants indirectly through this procedure, then they satisfy an integrality and a (genus) finiteness condition.

The integrality conjecture has been resolved by Ionel and Parker. A key innovation of their proof is the introduction of the cluster formalism: an ingenious device to side-step questions regarding multiple covers and super-rigidity. Their argument could not resolve the finiteness conjecture, however. The reason for this is that it relies on Gromov's compactness theorem for pseudo-holomorphic maps which requires an a priori genus bound. It turns out, however, that Gromov's compactness theorem can (and should!) be replaced with the work of Federer–Flemming, Allard, and De Lellis–Spadaro–Spolaor. This upgrade of Ionel and Parker's cluster formalism proves both the integrality and finiteness conjecture.

This talk is based on joint work with Eleny Ionel and Aleksander Doan.

Abstracts for the Mini Talks

Mini talk: Ian Hambleton (University of Ontario)

A Stability Range for 4-Manifolds

We introduce a new stable range invariant for the classification of closed, oriented topological 4–manifolds (up to s-cobordism), after stabilization by connected sum with a uniformly bounded number of copies of $S^2 \times S^2$

Mini talk: Alfred Holmes (Oxford University)

Spin(7) Instantons from the ADHM Seiberg Witten Equations

I will outline a proposal for constructing Spin(7) instantons on the positive spinor bundle of a four manifold from solutions to the ADHM Seiberg Witten equations.

Mini Talk: Partha Sarathi Ghosh (Université libre de Bruxelles)

A six dimensional version of the Seiberg-Witten equations

This is to introduce a new set of gauge-theoretic equations for a closed six-dimensional manifold which are influenced by the well known Seiberg-Witten equations in dimension four, this potentially can lead to a smooth-invariant. I also describe some ansatz, which are essential for understanding the geometry of the moduli space

Mini Talk: Jiakai Li (Harvard University)

Monopole Floer homology and real structures

I will explain how real structure appears in Seiberg-Witten theory for manifolds with involutions. In particular, I will define a "real" version of Kronheimer-Mrowka's monopole Floer homology. As a special case, we obtain Floer homology groups for links via their double branched covers equipped with covering involutions, which satisfy a skein exact triangle.

Mini Talk: Sudipta Ghosh (MPIM Bonn)

$SL(2, \mathbb{C})$-representations of homology $\mathbb{RP}^3$

Building on non-vanishing theorems of Kronheimer and Mrowka in instanton Floer homology, Zentner proved that if Y is a homology 3-sphere other than S^3, then its fundamental group admits a homomorphism to $SL(2, \mathbb{C})$ with non-abelian image. In this talk, we will discuss this result when Y is different from $\mathbb{RP}^3$ and $H_1(Y) = \mathbb{Z}/2$. This is joint work with Steven Sivek and Raphael Zentner

Mini Talk: Gard Olav Helle (University of Southern Denmark)

Computations of equivariant instanton Floer groups for spherical space forms

The spherical space forms are the orbit spaces obtained from a free and linear action of a finite subgroup of SO(4) on the 3-sphere. For this family of 3-manifolds one may try to compute equivariant instanton Floer groups directly with the aid of an equivariant version of the ADHM correspondence. In this talk I will describe the key ideas involved in this, how they led to complete calculations in the case of finite subgroups of SU(2) and work in progress toward computations for the remaining spherical space forms

Mini talk: Juan Munoz-Echániz (Columbia University)

Dehn twists on contact 3-manifolds and monopole Floer homology

I will describe the first known examples of exotic contactomorphisms in dimension 3 with infinite order in the contact mapping class group. These are given by certain Dehn twists along embedded spheres. The main tool that we use to detect this phenomenon is a families generalisation of the Kronheimer–Mrowka contact invariant in monopole Floer homology.

Mini Talk: Christopher Herald (University of Nevada at Reno)

The endomorphism of the wrapped Fukaya category of the pillowcase induced by the earring tangle

The singular instanton homology of a knot, defined by Kronheimer and Mrowka, involves $SO(3)$ connections that are singular along the knot and along a small meridian to the knot, on an $SO(3)$ bundle that is nontrivial on each surface crossing an arc connecting the knot to the meridian. The flat moduli space of such connections may be identified with an appropriately defined traceless $SU(2)$ character variety of the knot, meridian and arc connecting them (the "knot with an earrin"). If the knot is decomposed into two 2-tangles along a Conway sphere, one adorned with the earring, this flat moduli space may be identified with a fiber product of two traceless character varieties of the two tangles.

In joint work with Cazassus, Kirk and Kotelskiy, we showed that adding the earring to a tangle has the effect on the traceless character variety of composing with a specific Lagrangian correspondence, given by immersed genus three surface in the product of two pillowcases. Here, the pillowcase refers to the traceless character variety of the 4-punctured Conway sphere. We identified the immersed surface up to homotopy to obtain a description of the induced endomorphism of the wrapped Fukaya category of the pillowcase.

In subsequent work with Paul Kirk, we identify the immersed surface up to regular homotopy as a Lagrangian bifold immersion. We also show that an alternative marking, namely connected sum with a theta curve, alters the traceless character variety by a strikingly similar Lagrangian correspondence.