Assignment 1

Due date: Wednesday, 20 October (8pm)

Ex 2

This exercise explores the scale invariance of the KdV equation.

1. Show that if $u(x,t) = g(x,t)$ solves the KdV equation
$$u_t + 6uu_x + u_{xxx} = 0 \ ,$$
so does $u(x,t) = Ag(Bx, Ct)$, provided that the constants B and C are related to A in a way which you should determine. [15 marks]

2. Apply this transformation to the basic KdV solution
$$u(x,t) = \frac{2}{\cosh^2(x-4t)}$$
to construct a one-parameter family of one-soliton solutions of the KdV equation. [10 marks]

3. Find a formula relating the velocities to the heights for solitons in this one-parameter family. How does the spatial width of a soliton in this family change if its velocity is rescaled by a factor of 4? [10 marks]

Ex 4

Consider a pair of solitons with velocities m and n in the ball and box model, with $m > n$ and the faster soliton to the left of the slower one, with separation $l \geq n$ (*i.e.* there are l empty boxes between the two solitons). Evolve various such initial conditions forward in time using the ball and box rule, for different values of m, n and l. Prove that the system always evolves into an oppositely-ordered pair of the same two solitons, and find a general formula for the phase shifts of the solitons in terms of m and n. [45 marks]

Ex 11

Find the dispersion relation and the phase and group velocities for:

(a) $$u_t + u_x + \alpha u_{xxx} = 0$$ [10 marks]

(b) $$u_{tt} - \alpha^2 u_{xx} = \beta^2 u_{ttxx}$$ [10 marks]