Multiple M2-branes and Supergravity background fields

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String theory, M-theory and and supergravity

- String Theory
  - D-branes
  - 10D Supergravity

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- M-theory
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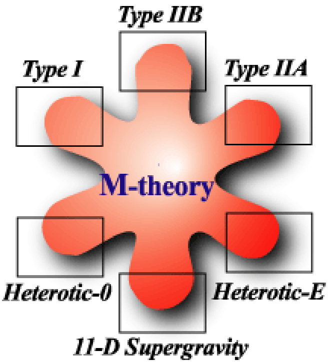
  - 11D Supergravity



D-branes

- For a single D2-brane the (bosonic) action:

$$\mathcal{L} = -\frac{1}{(2\pi\alpha')^2 g_{YM}^2}\sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}.$$

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- For N coincident D-branes the (low energy) action is:

$$\mathcal{L} = \frac{1}{g_{YM}^2}\,\mathrm{Tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D_\mu X^i D^\mu X^i + V(X) - i\bar{\Psi}\Gamma^\mu D_\mu \Psi - i\bar{\Psi}\Gamma^i[X^i, \bar{\Psi}]\right),$$

where

$$V(X) = \frac{1}{4}[X^i, X^j][X^i, X^j].$$

Background fields

- In type IIA string theory and 10D supergravity we have anti-symmetric C fields: C_a, C_{abc}, C_{abcde}, etc.
- Each of these couples naturally to a single D-brane,

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- With multiple D-branes, we need a more complicated coupling to preserve T-duality,

$$S = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda \mathbf{i}_X \mathbf{i}_X}\left(\sum C_{(n)} e^B\right)\right] e^{\lambda F}\right).$$

For D2-branes, this will include terms like $C_{(3)}$, $X\,X\,C_{(5)}$, $X\,X\,C_{(3)} \wedge B$, etc.

Branes in M-theory

- For an M2-brane action we need:
  - 16 supersymmetries,
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  - degrees of freedom to grow as $N^{\frac{3}{2}}$.

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- For an M2-brane action we need:
  - 16 supersymmetries,
  - 8 scalars,
  - degrees of freedom to grow as $N^{\frac{3}{2}}$.
- An important clue was provided by the Basu-Harvey equation,

$$\partial_\mu X \propto [X, X, X].$$

Bagger and Lambert (and separately Gustavson) were motivated by this to come up with an action based on a *three algebra*.

Three algebras

Generalize the idea of a Lie algebra to a **three algebra**.

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\begin{aligned}
[X, Y] \quad & \rightarrow \quad [X, Y, Z] \\
[T^a, T^b] = f^{ab}{}_c T^c \quad & \rightarrow \quad [T^a, T^b, T^c] = f^{abc}{}_d T^d.
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We also have

- Generalised Jacobi Identity
- A Trace (Tr)
- A metric $h^{ab} = \mathrm{Tr}(T^a, T^b)$

The BLG Action

Using the three algebra, we can write down the following action[1] describing multiple M2-branes:

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{2}(D_\mu X^I)(D^\mu X_I) + \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu \Psi - \frac{i}{4}[\bar{\Psi}, X^I, X^J]\Gamma_{IJ}\Psi\right) \\ - V + \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}),$$

where

$$D_\mu X_a = \partial_\mu X_a - \tilde{A}_\mu{}^b{}_a X_b, \quad \tilde{A}_\mu{}^b{}_a = f^{cdb}{}_a A_{\mu cd},$$

$$V = \frac{1}{12}\,\mathrm{Tr}\left([X^I, X^J, X^K][X^I, X^J, X^K]\right)$$

[1] Bagger, Lambert (2007) [hep-th:0711.0955]

The ABJM action

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The ABJM action

- Aharony, Bergman, Jafferis, and Maldacena wrote down another candidate action based with a $U(N) \times U(N)$ gauge symmetry.
- This can be written in terms of a similar three-algebra structure but without the full anti-symmetry:

$$
\begin{aligned}
\mathcal{L} = &- \operatorname{Tr}\left(D^\mu \bar{Z}_A D_\mu Z^A + i\bar{\Psi}^A \Gamma^\mu D_\mu \Psi_A\right) - V \\
&- i\operatorname{Tr}\left(\bar{\Psi}^A[\Psi_A, Z^B; \bar{Z}_B] - 2\bar{\Psi}^A[\Psi_B, Z^B; \bar{Z}_A]\right) \\
&+ \frac{i}{2}\varepsilon^{ABCD}\operatorname{Tr}\left(\bar{\Psi}_A[Z_C, Z_D; \psi_B] - \bar{Z}_D[\bar{\psi}_A \psi_B; \bar{Z}_C]\right) \\
&+ \frac{k}{4\pi}\operatorname{Tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3}\tilde{A} \wedge \tilde{A} \wedge \tilde{A}\right)
\end{aligned}
$$

$$
Z^A = X^A + iX^{A+4}
$$

Back to D2-branes

- We want to be able to recover the multiple D2-brane action from the multiple M2-action

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  - Split up the gauge field(s) into A^- and A^+. Then A^- becomes an auxilary field and can be integrated out.
  - Remaining gauge field 'eats' the Goldstone boson and acquires the necessary degrees of freedom.
  - In the limit of $v \to \infty$, we recover the multiple D2-brane action!

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Coupling to background fields

- 11D supergravity has a 3-form C_{abc} and a 6-form C_{abcdef} that we'd like to include in the M2 action.

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- The following is a candidate coupling[3],

$$\ldots + \varepsilon^{\mu\nu\rho} \operatorname{Tr}\left(C^{(1)}_{\mu A \bar{B}} D_\nu Z^A D_\rho Z^\dagger_B \right) + \ldots$$

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- This correctly reproduces some of the expected terms in the D2 action:

$$\ldots + \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(C_{\mu ij} D_\nu X^i D_\rho X^j\right) + \ldots$$

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Current and future work

- Look at a $C_{(3)} \wedge C_{(3)}$ term to hopefully get a $C_{(3)} \wedge B$ term in the D2 action,
- Reduce this form of the action back to the BLG action,
- Consider supersymmetry of this new action.