Response from probing black holes

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Aims

- Compute retarded Green's functions in strongly-coupled field theories using AdS/CFT.
- Show off some pretty pictures.

What is a retarded Green's function?

A retarded Green's function characterises the linear response of a field theory to an external perturbation:

$$\langle \mathcal{O}(k) \rangle = G_R(k) \phi_0(k), \quad k_\mu = (-\omega, \vec{k})$$

It gives you

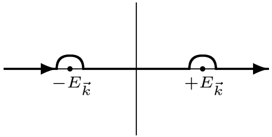
- the excitation spectrum;
- transport coefficients;
- the location of the Fermi surface;
- an indication of whether a state is unstable.

How are they calculated?

Two options:

1. With great difficulty.
2. Using AdS/CFT.

Come again?



Why is it hard to calculate G_R?

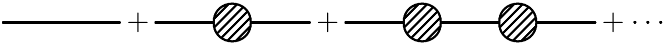
Free scalar field of mass m in d dimensional Minkowski:

$$G_R^{\text{free}}(k) = -\frac{1}{k^2 + m^2}$$

In a more general system:

- Don't know fundamental degrees of freedom.
- Strong coupling $\Rightarrow$ cannot use perturbation theory to find G_R^{full}.
- Non-trivial density matrix.

$G_F^{\text{full}} =$



What is AdS/CFT?

AdS/CFT is

- a compelling mathematical map between gravitational systems and field theories;
- ‘holographic’: the physics of a given region is fully encoded in the physics on the boundary of that region;
- a tool for exploring QFT at strong coupling.

What is AdS/CFT?

AdS/CFT is

- dangerous;
- not going to give us the Standard Model or a high-T_c superconductor anytime soon;
- a guide which could lead to general principles.

Using AdS/CFT: Correlation functions

Correlation functions are the basic objects in a QFT.

Original prescription[1] formulated in Euclidean signature:

$$
\begin{aligned}
Z_{\text{QFT}} &= Z_{\text{string}} \\
\log \left\langle e^{\int \phi_0 \mathcal{O}} \right\rangle &= S_{\text{o.s.}}\left[\phi(r,x) \to \phi_0(x)\right] \\
\Rightarrow \langle \mathcal{O}(x)\mathcal{O}(y)\rangle &= \frac{\delta^2 S_{\text{o.s.}}}{\delta\phi_0(x)\delta\phi_0(y)}
\end{aligned}
$$

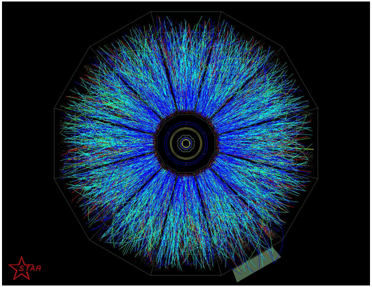
Each gravity field ϕ corresponds to some operator $\mathcal{O}$ in the boundary theory.

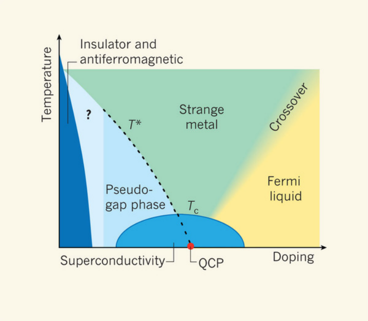
[1] hep-th/9802109, hep-th/9802150

Using AdS/CFT: Examples

Study behaviour at non-zero density, temperature and so on by modifying the gravitational background.

- Examples: quark-gluon plasma, strange metals.[2]





[2]Figures taken from damtp.cam.ac.uk (left) and from Nature 468, 184–185 (2010) (right).

Using AdS/CFT: Real-time processes

Need to do more if we want to study non-equilibrium behaviour.

- First step: linear response theory and G_R.
- Cannot just Wick rotate $\Rightarrow$ need a new prescription.

Using AdS/CFT: Extracting G_R

Simple case: scalar field ϕ of mass m in an AdS black hole in $(d{+}1)$ dimensions.[3]

1. Solve $(\nabla^2 - m^2)\phi = 0$ subject to infalling boundary conditions at the event horizon.
2. Asymptotic expansion:

$$\phi(r) \overset{r\to\infty}{\sim} \phi_0 r^{\Delta-d}(1+\cdots) + \langle\mathcal{O}\rangle r^{-\Delta}(1+\cdots)$$

3. Extract G_R using

$$\langle\mathcal{O}(k)\rangle = G_R(k)\phi_0(k)$$

Coupled perturbations are more complicated: matrix of correlators.

[3] hep-th/0205051, 0809.3808

Summary so far

- Leading behaviour encodes the source $\Rightarrow$ perturbation of field theory action.
- Subleading behaviour encodes the response $\Rightarrow$ how the field theory changes as a result.

Aside: Quasinormal modes

Characteristic oscillations of black objects.

- Impose infalling boundary conditions at the horizon.
- Non-Hermitian eigenproblem $\Rightarrow$ complex eigenfrequenices.
- Potential diverges at the boundary of AdS $\Rightarrow$ field must vanish there.

Poles in $G_R \longleftrightarrow$ Quasinormal frequencies[4]

$\Rightarrow$ Check on our results.

[4] hep-th/0112055, hep-th/0205051

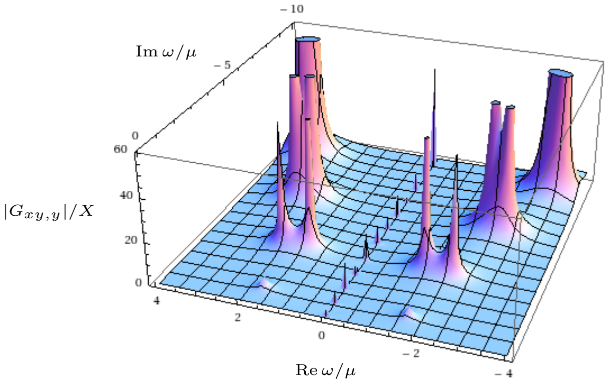
Shear channel correlators: Setup

Based on recent work with Daniel K. Brattan: 1012.1280

- Thermal field theory in (2+1)-dimensional Minkowski with $U(1)$ charge density.
- Gravitational dual is an Einstein-Maxwell theory.
- Boundary theory at equilibrium is dual to non-extremal RN-AdS$_4$.
- Coupled equations for metric and gauge field fluctuations.

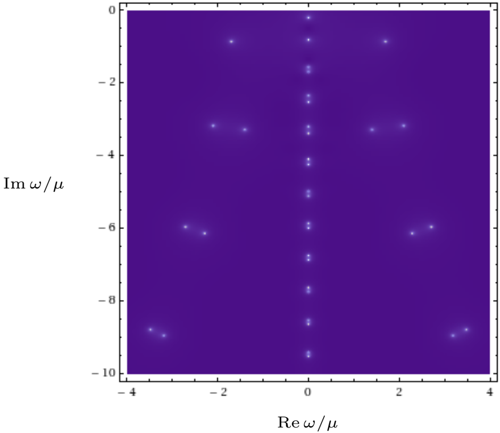
Shear channel correlators: Example

Compute G_R for conserved currents under fluctuations *transverse* to the direction of momentum flow.



Shear channel correlators: Check

The appropriate quasinormal frequencies:



Departure from hydrodynamics

Most studies restrict to long wavelengths and/or small frequencies.

$\Rightarrow$ Effective description: hydrodynamics.

- Use perturbative expansion in small $\omega, \vec{k}$ to find transport coefficients, e.g.

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} G_{xy,xy}(\omega, \vec{k} = 0)$$

- Only leading poles are significant and their locations give the dispersion relation, e.g.

$$\omega = -iD\vec{k}^2$$

Our results $\Rightarrow$ probe deeper into the dynamics.

Pole dancing

Coupled perturbations, many correlators, all built from $\hat{\Pi}_+$ and $\hat{\Pi}_-$.

Study $\hat{\Pi}_-$: it contains the (single) 'hydrodynamic' pole.

- Fix T/μ and increase k/μ: click here.
- Fix k/μ and increase T/μ: click here.

Axis motion

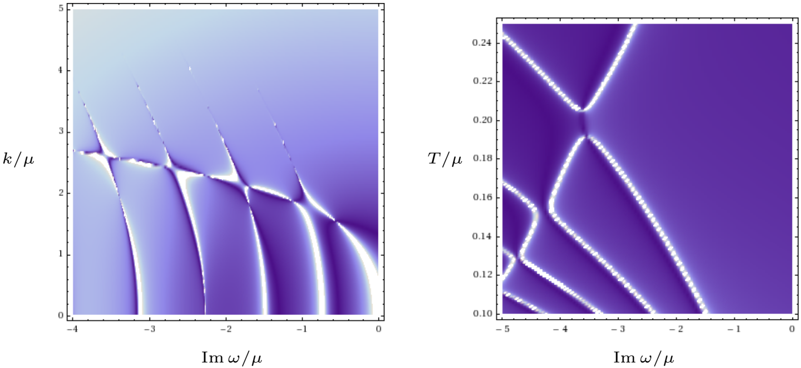


Figure: Motion of on-axis poles as k/μ is varied (left) or as T/μ is varied (right).

Bottoms up

Whopping caveats:

- Dual theory may not be well-defined!
- May miss low temperature behaviour.

Which features are generic?

Summary and outlook

- Use AdS/CFT to extract retarded Green's functions for strongly-coupled field theories.
- Rich dynamics beyond the hydrodynamic regime.
- Stepping stone to more 'realistic' setups.

Bonus

A consistent truncation of type IIB supergravity:[5]

$$
\begin{aligned}
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \Bigg[R - \frac{L^2}{3} F_{\mu\nu} F^{\mu\nu} \\
+ \left(\frac{2L}{3}\right)^3 \frac{1}{4} \epsilon^{\lambda\mu\nu\sigma\rho} F_{\lambda\mu} F_{\nu\sigma} A_\rho - \frac{1}{2} \Bigg(\partial_\mu \eta \partial^\mu \eta \\
+ \sinh^2 \eta \left(\partial_\mu \theta - 2A_\mu\right) \left(\partial^\mu \theta - 2A^\mu\right) \\
- \frac{6}{L^2} \cosh^2 \frac{\eta}{2} \left(5 - \cosh \eta\right) \Bigg) \Bigg]
\end{aligned}
$$

where $\epsilon^{01234} = 1/\sqrt{-g}$ and $F = dA$. The complex scalar has modulus η and phase θ.

Motion of QNF for η: click here.

[5] 0907.3510