Broken Baby Skyrmions

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The stability of particles in our theory arise from their topological structure.

Hence they cannot decay into topologically trivial fields.

The theory is classified using homotopy classes and topological charge.

Theorem

Suppose that for an arbitrary, finite energy field configuration $\phi(\mathbf{x})$, which is not the vacuum, the function $e(\mu)$ has no stationary point. Then the theory has no static solutions of the field equation with finite energy, other than the vacuum.

Lumps

Overcome Derricks theorem in 2-dim by having no potential term such that $e(\mu) = E_2$ and is independent of μ and giving our solution conformal invariance.

Strictly speaking these are not Solitons due to their scale instability, hence known as lumps.

lagrangian

$$\mathcal{L} = \frac{1}{4}\partial_\mu \phi \cdot \partial^\mu \phi + \nu (1 - \phi \cdot \phi)$$

The target space is the unit 2-sphere S^2, which we parameterise as ϕ being a three-component vector with $\phi \cdot \phi = 1$.

Lumps

At spatial infinity ϕ must tend to a constant vector taken to be $\phi_\infty = (0,0,1)$. Compactifying our space to $\mathbb{R}^2 \cup \{\infty\} \equiv S^2$.

ϕ is now a map $\phi : S^2 \to S^2$ with homotopy group $\pi_2\left(S^2\right) = \mathbb{Z}$.

When numerically simulated this leads to the solutions contracting to a point or expanding indenfinatley.

We want to break this conformal invariance by introducing new terms.

We can aquire this new term by considering the Skyrme model to create a planar version of the theory.

Skyrme Model

The Skyrme Model is an extension of the Sigma model in 3-dim.

lagrangian

$$\mathcal{L} = \frac{F_\pi^2}{16} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2} Tr\left(\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]\left[\partial^\mu U U^\dagger, \partial^\nu U U^\dagger\right]\right)$$

where $U(t, \mathbf{x})$ is an $SU(2)$-valued scalar.

The Skyrme model is a model of Baryons which arise as topological solitons, with baryon number $\leftrightarrow$ topological charge.

This model does produce nuclei solutions.

$$e(\mu) = \frac{1}{\mu} E_2 + \mu E_4$$

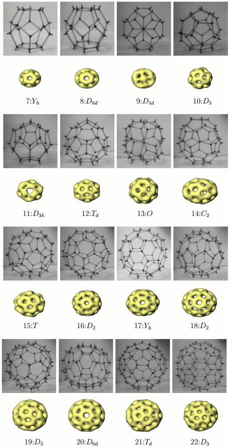


Figure: Baryon density isosurfaces for various charges

The Skyrme Model

- The Skyrme model is a nonlinear field theory in $(3+1)$ dimensions that admits topological solutions.
- These solutions can be interpreted as baryons.
- The number of colours N seems to play no role in the classical model.
- It is interesting to consider an N that plays a larger role in the classical soliton solution.

Baby Skyrmions

Can stabalise our lumps using a planar analogue of the Skyrme term.

lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi + \frac{\kappa^2}{4}\left(\partial_\mu\phi \times \partial_\nu\phi\right) \cdot \left(\partial^\mu\phi \times \partial^\nu\phi\right) - V(\phi)$$

The topological classification is the same as the $O(3)$ sigma model.

$$e(\mu) = E_2 + \mu^2 E_4 + \frac{E_0}{\mu^2}$$

The Baby Skyrme Model

- The Baby Skyrme model is a $(2+1)$ dimensional analogue to the Skyrme Model.
- It can be used as a toy model for testing ideas regarding the full Skyrme model.
- It does however have applications in condensed matter physics.
- Can we introduce a potential that breaks the standard $O(3)$ symmetry to produce a theory of N partons?

The Standard Potential

The standard potential is aquired from the conventional pion mass term from the full Skyrme model.

Standard Potential

$$V(\phi) = m^2 (1 - \phi_3)$$

This gives an axially symmetric single soliton with the scale of the soliton given by $\sqrt{\kappa/m}$.

We will need to break this symmetry if we want to introduce partons into our model.

For N partons the natural choice for our new symmetry is the Dihedral group D_N.

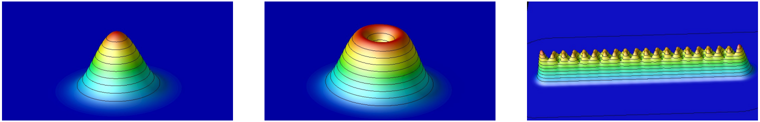


Figure: Standard Baby Skyrmion solutions for $B = 1, 2, 20$

Variations on the Standard Potential

The majority of variations retain the global $\mathcal{O}(2)$ symmetry with a few exceptions,

easy plane potential

$$V = \frac{1}{2} m^2 \phi_1^2$$

J. Jaykka and M. Speight, Phys. Rev. D82, 125030 (2010).

D_2 potential

$$V = \frac{1}{2} m^2 \left(1 - \phi_3^2\right) \left(1 - \phi_1^2\right)$$

R. S. Ward, Nonlinearity 17, 1033 (2004).

A New Potential

Broken Potential

$$V = m^2 \left|1 - (\phi_1 + i\phi_2)^N\right|^2 (1 - \phi_3)$$

J. Jaykka, M. Speight and P. Sutcliffe, Proc. Roy. Soc. Lond. A 468, 1085 (2012).

Breaks the global $O(3)$ symmetry to the dihedral group D_N, generated by the rotation and reflection

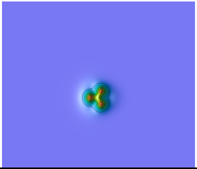
$$(\phi_1 + i\phi_2) \to (\phi_1 + i\phi_2)\, e^{\frac{i2\pi}{N}} \qquad (\phi_1, \phi_2, \phi_3) \to (\phi_1, -\phi_2, \phi_3)$$

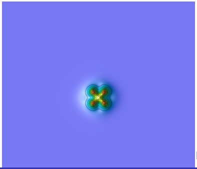
This gives the theory $N + 1$ vacua, the N roots of unity on the equator $\phi_3 = 0$ and $\boldsymbol{\phi} = (0, 0, 1)$.

Statics $B = 1$

- A gradient flow method was employed with parameters, $\kappa = m = 1$ and $\Delta x = 0.04$ with 501^2 grid points.
- Sutcliffe et al. produced the single soliton and multi-soliton solutions for $N = 3$ which our results agree with.
- Numerics shows the single soliton has the maximal symmetry D_N and is formed of N partons.

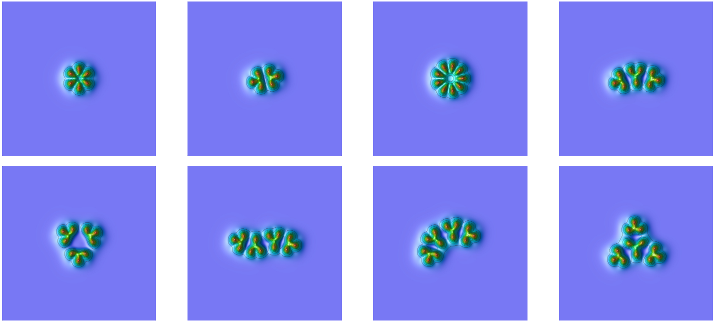
Table: Energy density plot of the single soliton solution for $N = 3$ (left) and $N = 4$ (right):





$N = 3, B \geq 2$

Table: Multi-soliton solutions, the top row shows $B = 2, 3$ and the bottom row shows $B = 4$



There appears to be two classes of solution above, classified by their symmetries:

- The maximal symmetry D_{3B} is formed of $3B$ partons located at the verticies of a regular $3B$-gon.
- The lower symmetry solutions are formed of B single solitons bound together.

Table: $N = 3$ solution energies

B	Icon	G	E/B
1		D_3	34.56
2		D_6	32.82
2		D_2	32.85
3		D_9	33.45
3		D_1	32.62
3		D_3	32.65
4		D_2	32.47
4		D_1	32.52
4		D_3	32.78

For $B > 2$ the maximal symmetry solutions are unstable to perturbations.
For $B = 2$ the hexagon solution is stable with an energy similar to the alternative bound solution such that it is difficult to suggest which one is the global minima.

Polyominos

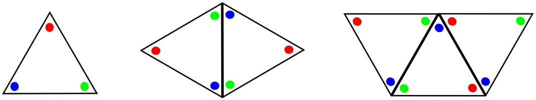
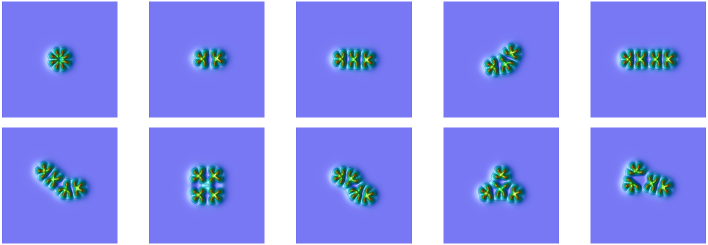


Figure: The colour configuration of polyiamonds upto $B = 3$

- We represent each $B = 1$ soliton as an equalateral triangle with each vertex being labelled with a different colour.
- Our solutions start to form the possible shapes from connecting the triangles with adjacent dots having different colours but with similar colours as close as possible.
- These shapes are called polyiamonds.
- Does this polyform structure continue for higher N?

$N = 4, B \geq 2$

Table: Multi-soliton solutions, the top row shows $B = 2, 3$ and the bottom row shows $B = 4$



$N = 4, B \geq 2$

The $N = 4$ case also has a stable ring solution for $B = 2$ but not for $B > 2$.

The bound states form the basic polyomino shape but are slightly distorted.

This behaviour appears to continue for increasing N.

Table: $N = 4$ solution energies

B	Icon	G	E/B
1		D_4	34.35
2		D_8	32.57
2		D_2	32.87
3		D_2	32.44
3		D_1	32.55
4		D_2	32.27
4		D_1	32.35
4		D_4	32.56
4		D_2	32.49
4		D_1	32.70
4		D_1	32.50

Dynamics

We want to scatter the broken baby skyrmions.
Does the parton structure affect the standard scattering?
A Runge-Kutta 4th order method was employed with similar dx to the static case and $dt = \frac{dx}{4}$.

$N = 3, B = 2scattering$

$N = 3, B = 3 scattering$

Summary

- Our chosen potential breaks the global symmetry from $O(3)$ to D_N.
- Stable single solitons are formed of N partons.
- Bound states take the form of Polyforms.
- Scattering is unchanged for $B = 2$.