Domain Wall Skyrmions

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Work in progress, in collaboration with P. Sutcliffe

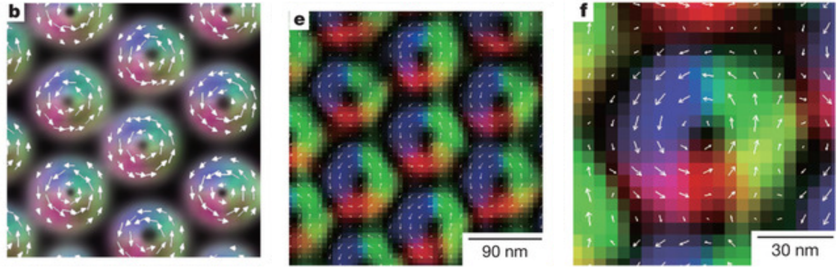
Outline

- Motivation
- Baby Skyrmions
- Skyrmion-Like Lumps from Domain Walls
- Statics
- Dynamics
- Conclusions and Outlook

Motivation

- Baby Skyrmions appear in Condensed Matter

[Yu, Onose et al. Nature 465, 901 (2010)]



- Domain Walls evident in lots of physical situations
  - Cosmology (Cosmic Strings)
  - Condensed matter
  - SUSY QCD

Motivation

- Computing applications of Domain walls– Racetrack memory

[Parkin, Roche et al., Magnetoelectronics and Spintronics and SpinAps]

Soliton Stability

- Consider time-independent fields. Then a field configuration which is a stationary point of the energy should be stationary against all variations.

Derrick's Theorem **[Derrick, J. Math. Phys. 5, 1252 (1964)]**

Suppose that for an arbitrary, finite energy field configuration $\Psi(\mathbf{x})$ which is not the vacuum, the function $e(\mu)$, the energy of the field configuration under rescaling $\mathbf{x} \to \mu\mathbf{x}$, has no stationary point. Then the theory has no static solutions of the field equation with finite energy, other than the vacuum.

Baby Skyrmion

- The theory is a (2+1)–dimensional model is described by

$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{\kappa^2}{4}(\partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi}) \cdot (\partial^\mu \boldsymbol{\phi} \times \partial^\nu \boldsymbol{\phi}) - m^2(1 - \phi_3),$$

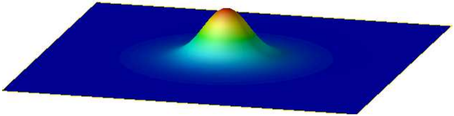
where $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ is a unit vector field.

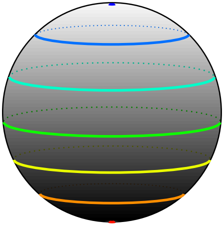
- Derrick's Theorem ensures the existence of static solitons
- Finite energy constraints mean $\boldsymbol{\phi} : S^2 \to S^2$ and has associated topological charge

$$B = -\frac{1}{4\pi}\int \boldsymbol{\phi} \cdot (\partial_1 \boldsymbol{\phi} \times \partial_2 \boldsymbol{\phi}) \,\mathrm{d}x_1 \,\mathrm{d}x_2,$$

Baby Skyrmion

- We find that the covering of the sphere is in the following way





Domain Wall solutions

- Consider a theory described by the

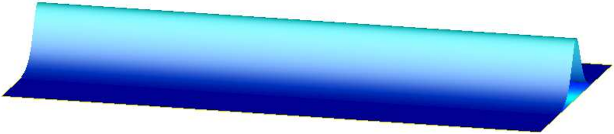
$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{m^2}{2}(1 - \phi_3^2).$$

for $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ a unit vector field.

- We see we obtain a domain wall connecting from vacua related to $\phi_3 = \pm 1$.

Domain Wall solutions

- Plotting the energy density (with $m = 1$) we find



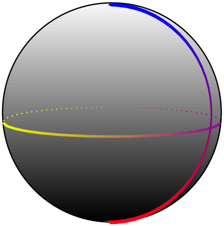
Skyrmion-like lumps from Domain Walls

- Now consider a theory described by

$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{m^2}{2}(1 - \phi_3^2) + \frac{m^2 g}{2}{\phi_2}^\ell.$$

for $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ a unit vector field, $g \in (0, 1)$, and with $\ell \in 2\mathbb{Z} + 1$ fixed.

- Covering of the sphere. Traverse between the vacua $\phi_3 = \pm 1$, passing through the vacua of the other term of the potential.



Skyrmion-like lumps from Domain Walls

- For the $\ell = 1$ case **[M. Nitta arXiv:1211.4916, 1302.0989v1]** we find that the minima of the potential is not at the poles of the target S^2, but actually at $\phi = (0, \frac{g}{2}, \pm\sqrt{1 - g^2/4})$.
- We choose to take $\ell = 3$, which avoids this problem, with the poles of the unit sphere being the true vacua of the model. For the remainder of this talk we shall take $\ell = 3$.

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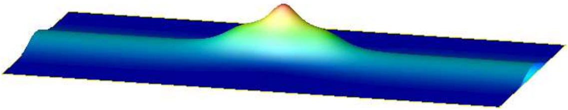
Static Solutions

- Static solution found by an energy minimisation algorithm (here with $m = 1$ and $g = 0.5$).



Skyrmion-like lumps from Domain Walls

- Now consider a theory described by

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for $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ a unit vector field, and with $g \in (0, 1)$.

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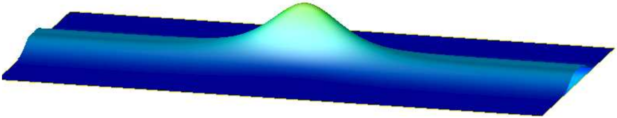
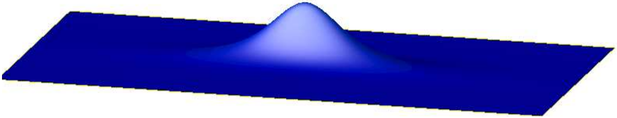


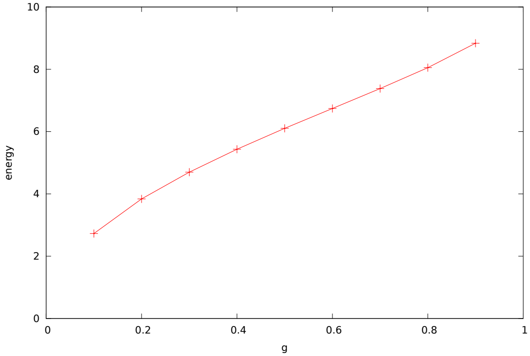
Static Solutions

- Charge density localised around the Skyrmion.



Parameter Dependence

- Can calculate the energy of the Skyrmion.
- The dependence of the energy varies with g as



Stability of Solution

- We place the Skyrmion on a perturbed domain wall and allow the system to evolve dynamically.

Soltion Scattering along wall

- We set up two Skyrmions on the domain wall and Lorentz boost them towards each other.
- v=0.1

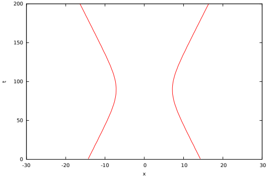
- We set up two Skyrmions on the domain wall and Lorentz boost them towards each other.
- v=0.2

Soltion Scattering along wall

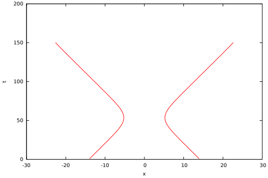
- We set up two Skyrmions on the domain wall and Lorentz boost them towards each other.
- v=0.4

Soltion Scattering along wall

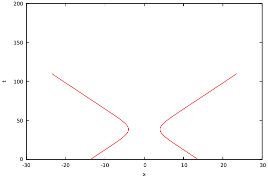
- Tracking the x–coordinate of the Skyrmions against time we find symmetric elastic collisions .



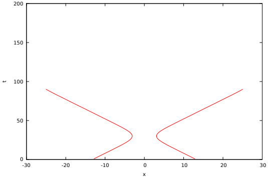
(a) v=0.1



(b) v=0.2



(c) v=0.3



(d) v=0.4

One dimensional dynamics

- Restrict to the one-dimensional system along the domain wall.

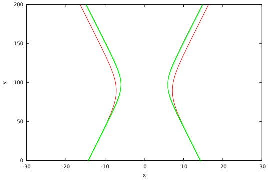
$$\mathcal{L} = \frac{1}{2}\partial_\mu \psi \partial^\mu \psi - \frac{m^2}{2}(1 - g\cos(\psi))$$

  where ψ is the one-dimensional field. The Lagrangian describes a sine-Gordon kink.

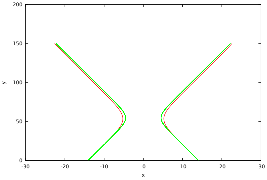
- We can find the solution of this analytically since this theory is integrable, with solution

$$\psi(x) = 4\arctan\left[\frac{v\sinh\left(\frac{\gamma\sqrt{m^2 g}x}{\sqrt{2}}\right)}{\cosh\left(\frac{\gamma v t\sqrt{m^2 g}}{\sqrt{2}}\right)}\right]$$

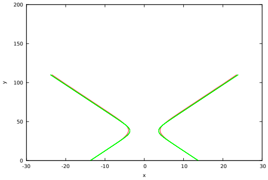
- We now plot the x–coordinate of the kink against time (green) to compare with the two dimensional case (red).



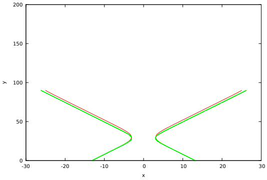
(a) v=0.1



(b) v=0.2



(c) v=0.3



(d) v=0.4

Other possible scatterings

- Multiple domain wall scattering
- Check: ADW, DW(S), ADW

Other possible scatterings

- Multiple domain wall scattering
- ADW(S), DW, ADW

Conclusion and Outlook

- Constructed Skyrmion-like lumps from domain walls
- Looked at their scattering behaviour
- Investigate the three–dimensional analogue of this model
- Impact of reintroducing Skyrme term
- Domain wall ring and investigate behaviour.