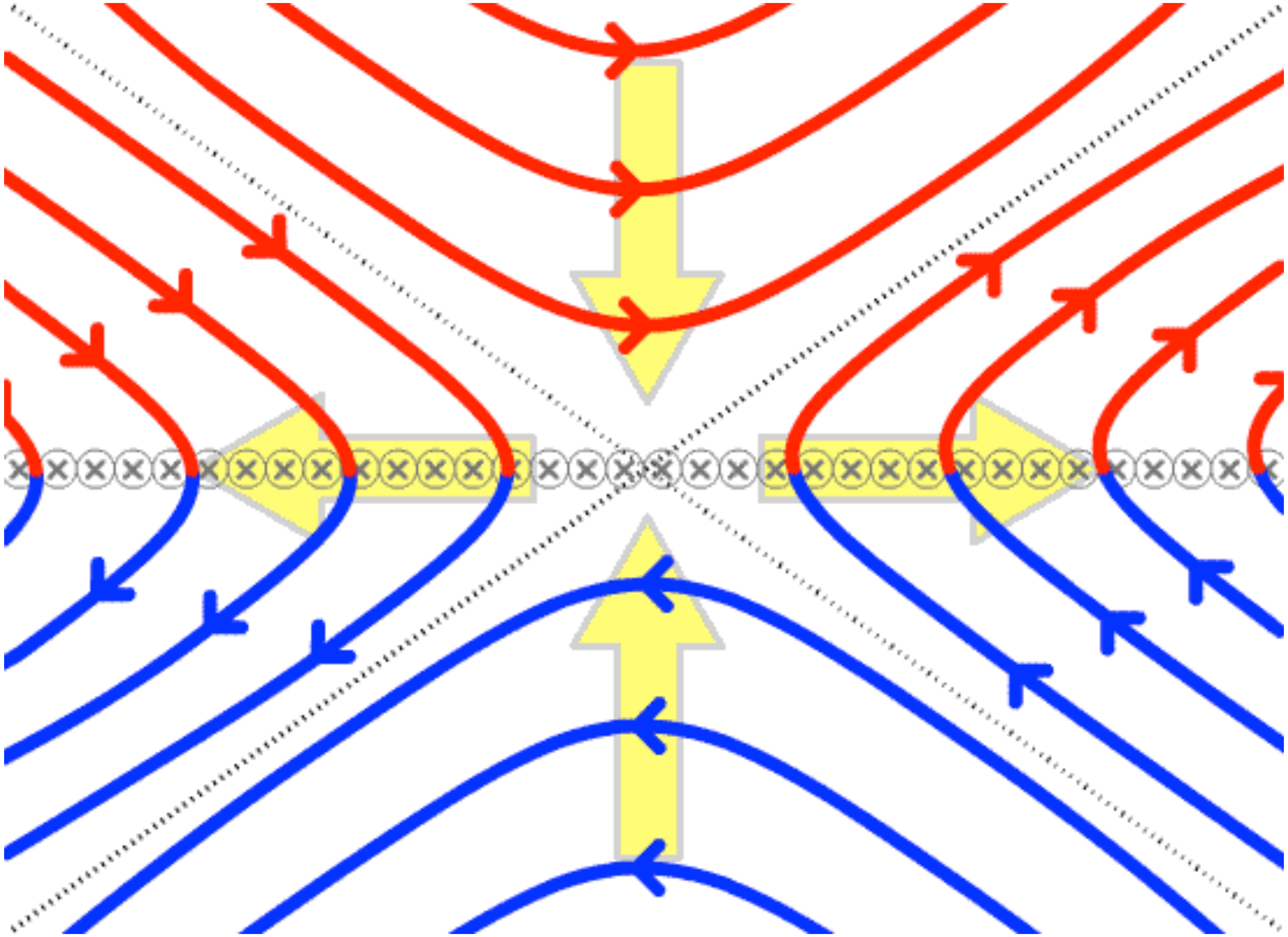
Quantifying Reconnection in Magnetic Flux Ropes

Anthony Yeates
with G. Hornig, A. Wilmot-Smith, A. Russell (University of Dundee)
& C. Prior (Durham University)

Flux rope workshop, UCLA, 12-Feb-2014



This is **not** typical of magnetic reconnection!

Magnetic reconnection - process by which a magnetic field in an almost-ideal plasma changes its topology (connectivity of magnetic field lines within domain or between boundaries).

Needs a non-ideal term in Ohm's Law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N} \quad \Longrightarrow \quad \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = -\nabla \times \mathbf{N}$$

Two types of reconnection:

1. $\mathbf{E} \cdot \mathbf{B} = 0$ **- 2D reconnection**
2. $\mathbf{E} \cdot \mathbf{B} \neq 0$ **- 3D reconnection**

Hornig & Schindler, *Phys. Plasmas* (1996)
Birn & Priest (eds.), *Reconnection of magnetic fields* (2007)

2D reconnection (E.B = 0)

Here

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{u} \times \mathbf{B}$$

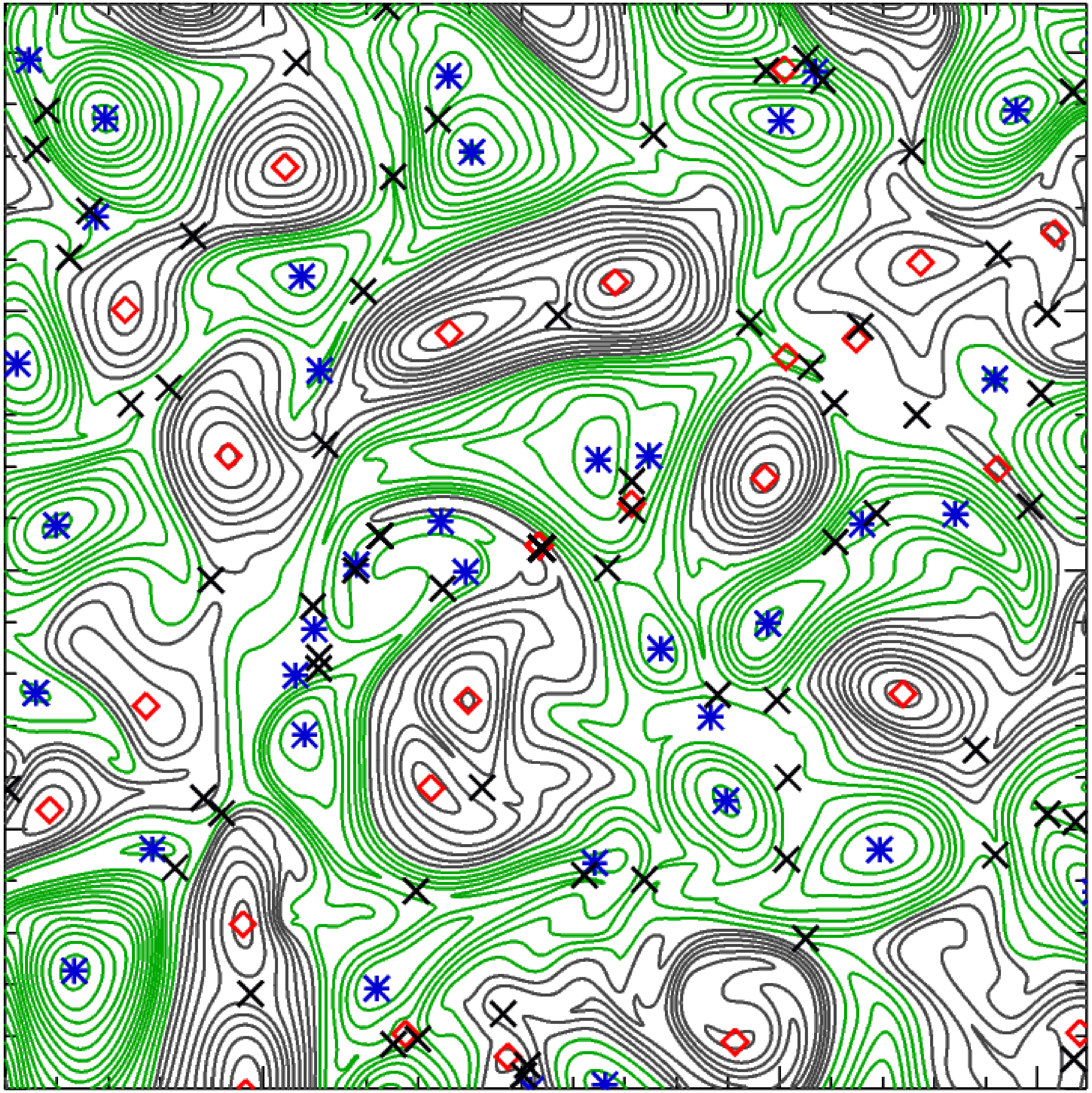
$$\Longrightarrow \quad \mathbf{E} + \mathbf{w} \times \mathbf{B} = 0$$

where **w** = **v** - **u** is a **field line velocity.**

We can write

$$\mathbf{w} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

so **2D reconnection occurs only at nulls** (**B** = 0) where **w** is singular.



Servidio et al., *Nonlin.Proc.Geophys.* (2011)

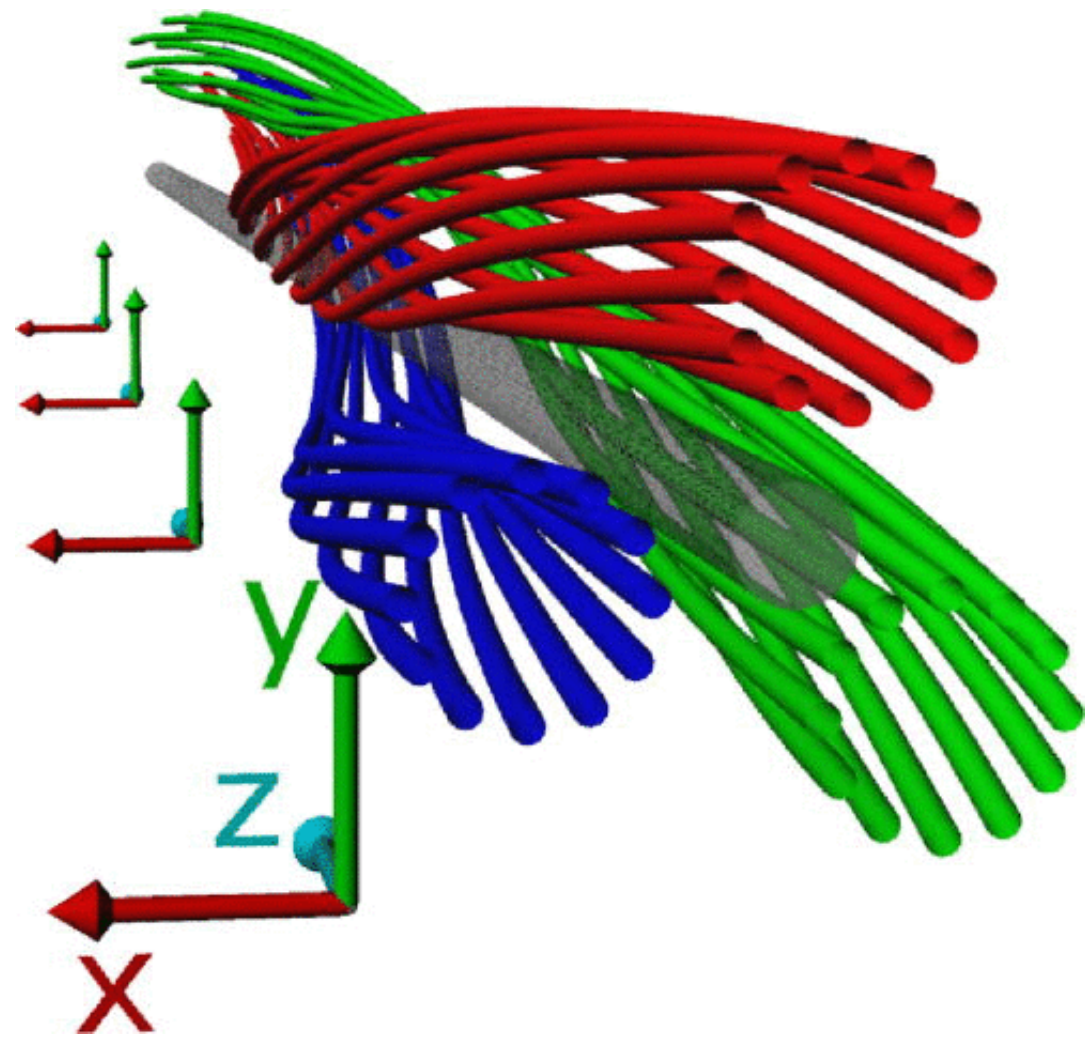
Flux ropes

- Strong guide field with no null points.

… so there is a field line velocity $\mathbf{w}$ with

$$\mathbf{E} + \mathbf{w} \times \mathbf{B} = \nabla\psi$$

- Changes in topology mean changes in field line connectivity *with respect to an ideal evolution*.

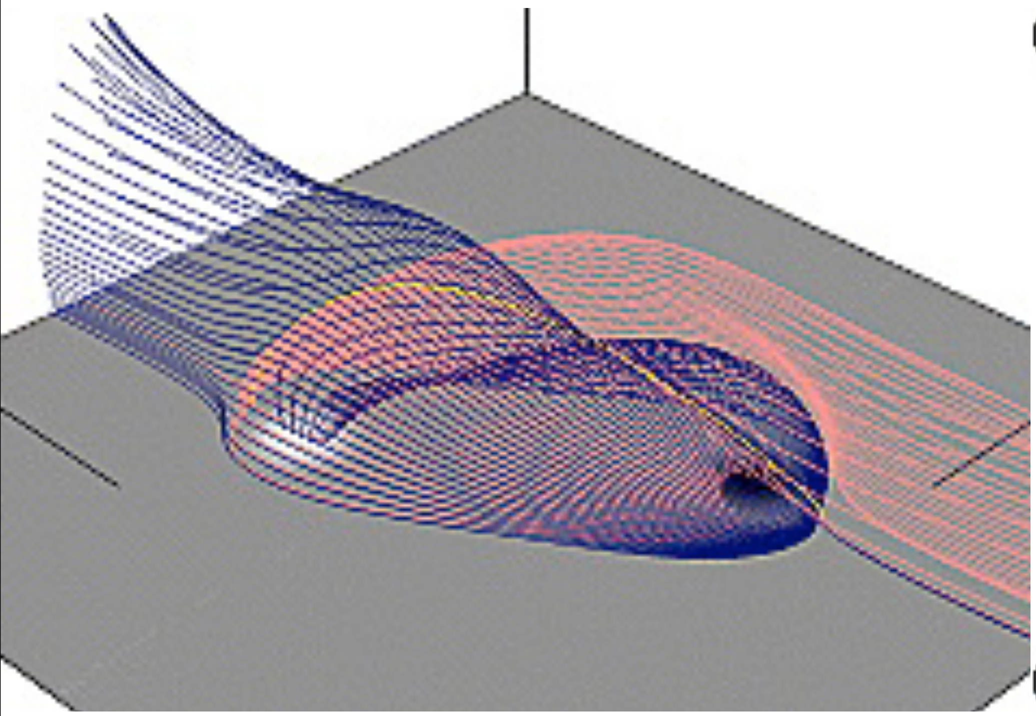


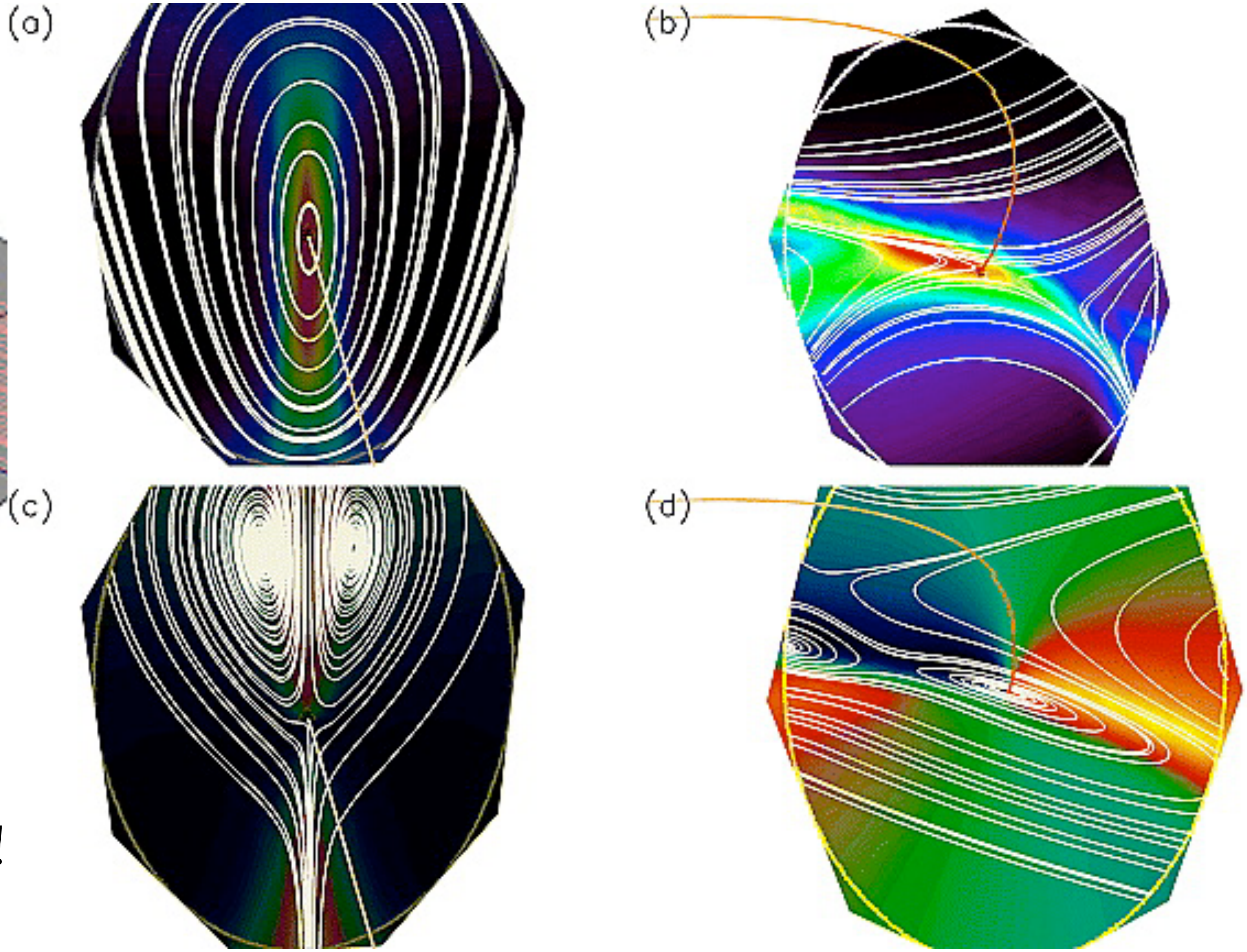
Gekelman et al, *Astrophys.J.* (2012)


DANGER!

Parnell et al., *J.Geophys.Res.* (2010)

"In 2D, magnetic topology and field line structure coincide, but in 3D global topological structures and local field structures do not coincide."

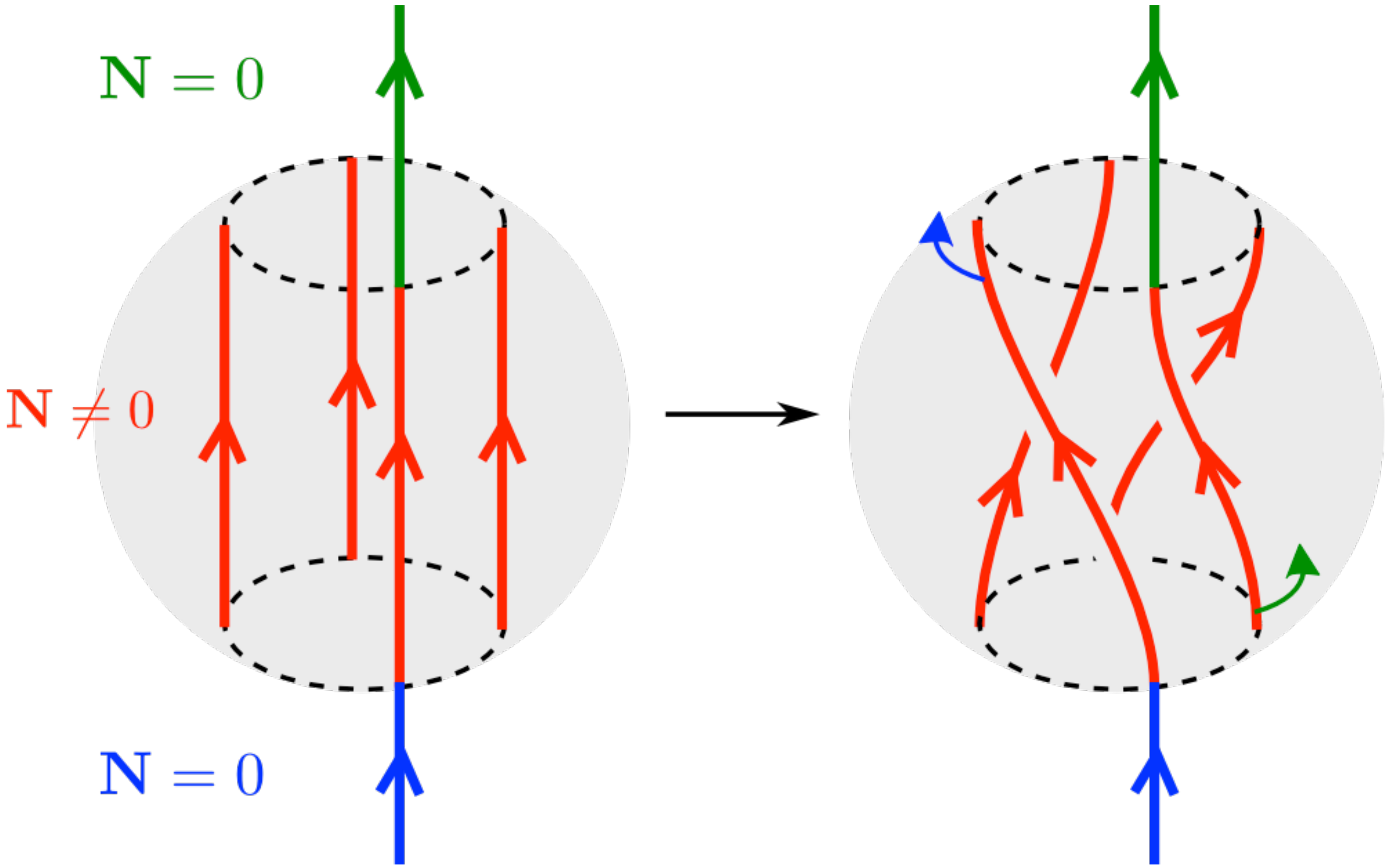




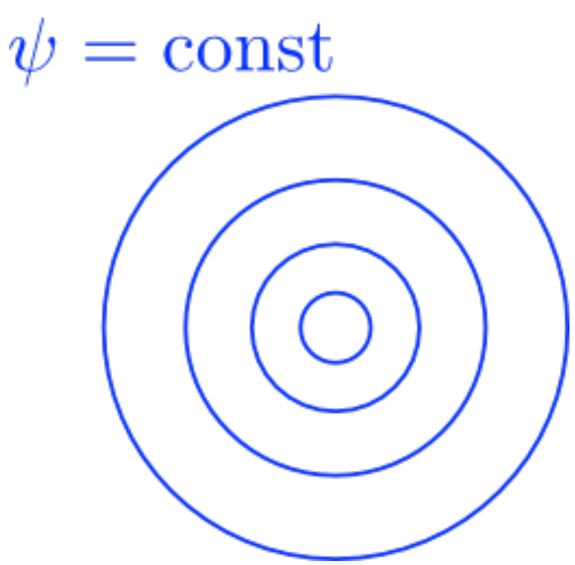
Cannot just look for local regions of X-type field lines!

Single reconnection site

Schindler et al., *J.Geophys.Res.* (1988); Hornig & Priest, *Phys. Plasmas* (2003)
Hesse et al., *Astrophys.J.* (2005)



$$\mathbf{E} + \mathbf{w} \times \mathbf{B} = \nabla\psi$$



- If $\mathbf{v} = 0$ on the boundary, then $\mathbf{w} \cdot \nabla\psi = 0$.
- For a localised non-ideal region, there is a clear reconnection rate.

Our approach

Quantify the global field structure.

- Need to measure changes in the field line connectivity.
- Characterize this with **ideal invariants**.

e.g. **magnetic helicity**

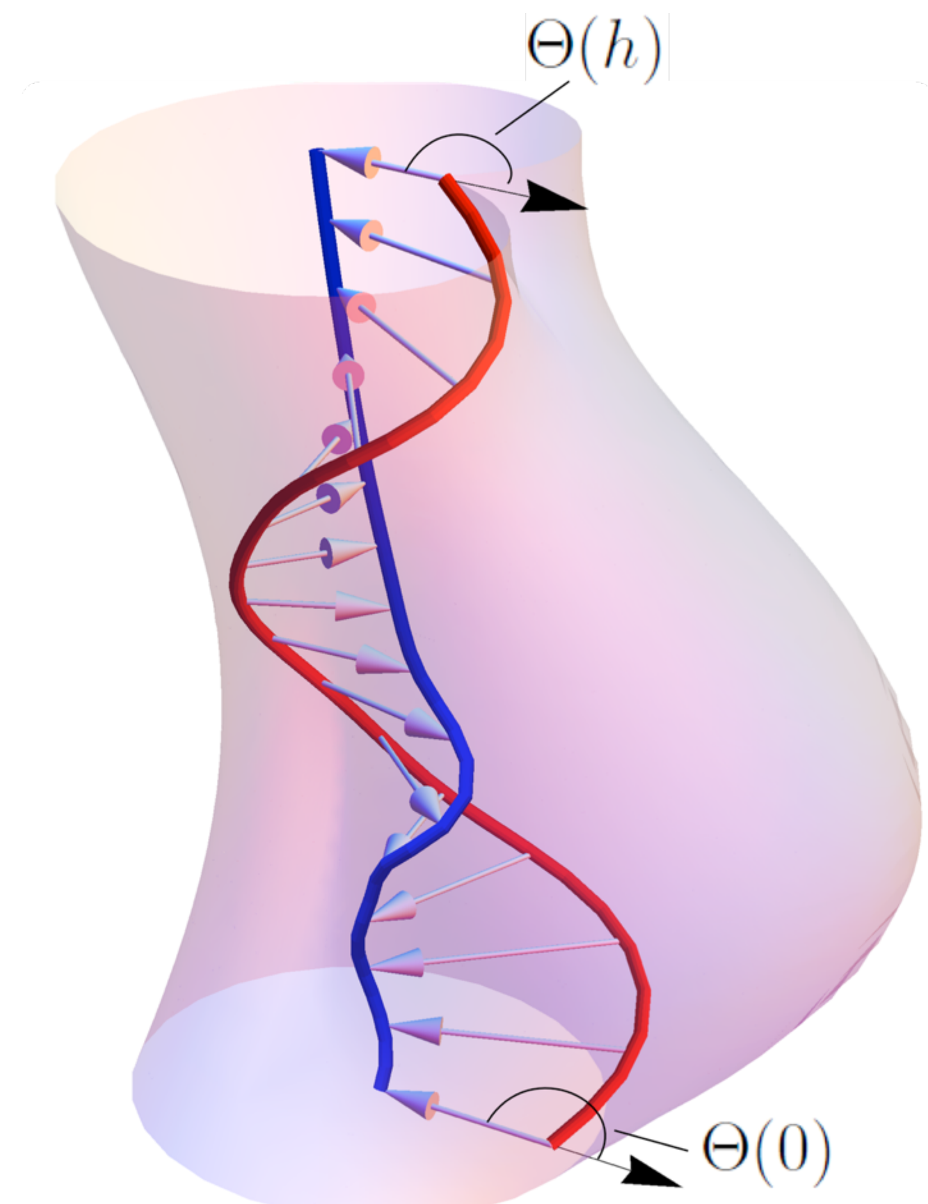
$$H = \int_V \mathbf{A} \cdot \mathbf{B} \, d^3x$$

In the **winding gauge**

$$\mathbf{A}(\mathbf{x}) = \frac{1}{2\pi} \int_{S_z} \frac{\mathbf{B}(\mathbf{y}) \times (\mathbf{x} - \mathbf{y})}{|\mathbf{x} - \mathbf{y}|^2} \, d^2y.$$

H is the average pairwise winding number between field lines.

Prior & Yeates (submitted)



Topological flux function

With **A** in winding gauge, define the **flux function**

$$\mathcal{A}(\mathbf{x}) = \int_{F_z(\mathbf{x})} \mathbf{A}\cdot\, d\mathbf{l}$$

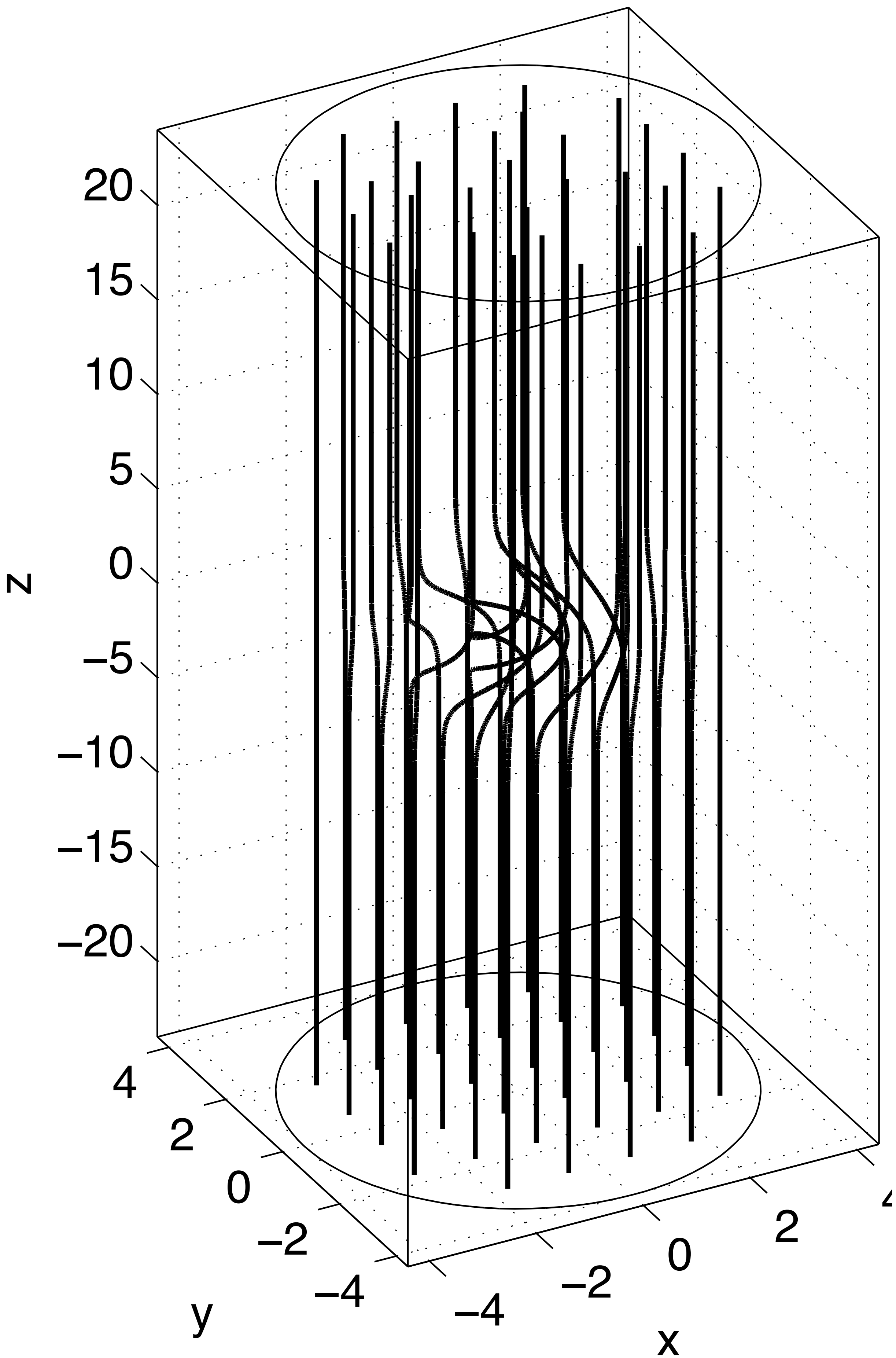
- This is the average winding number of **one** field line with all others.
- It is the "helicity per field line": Berger, *Astron.Astrophys.* (1988)

$$H = \int_{S_0} \mathcal{A}(\mathbf{x}) B_z(\mathbf{x})\, d^2x$$

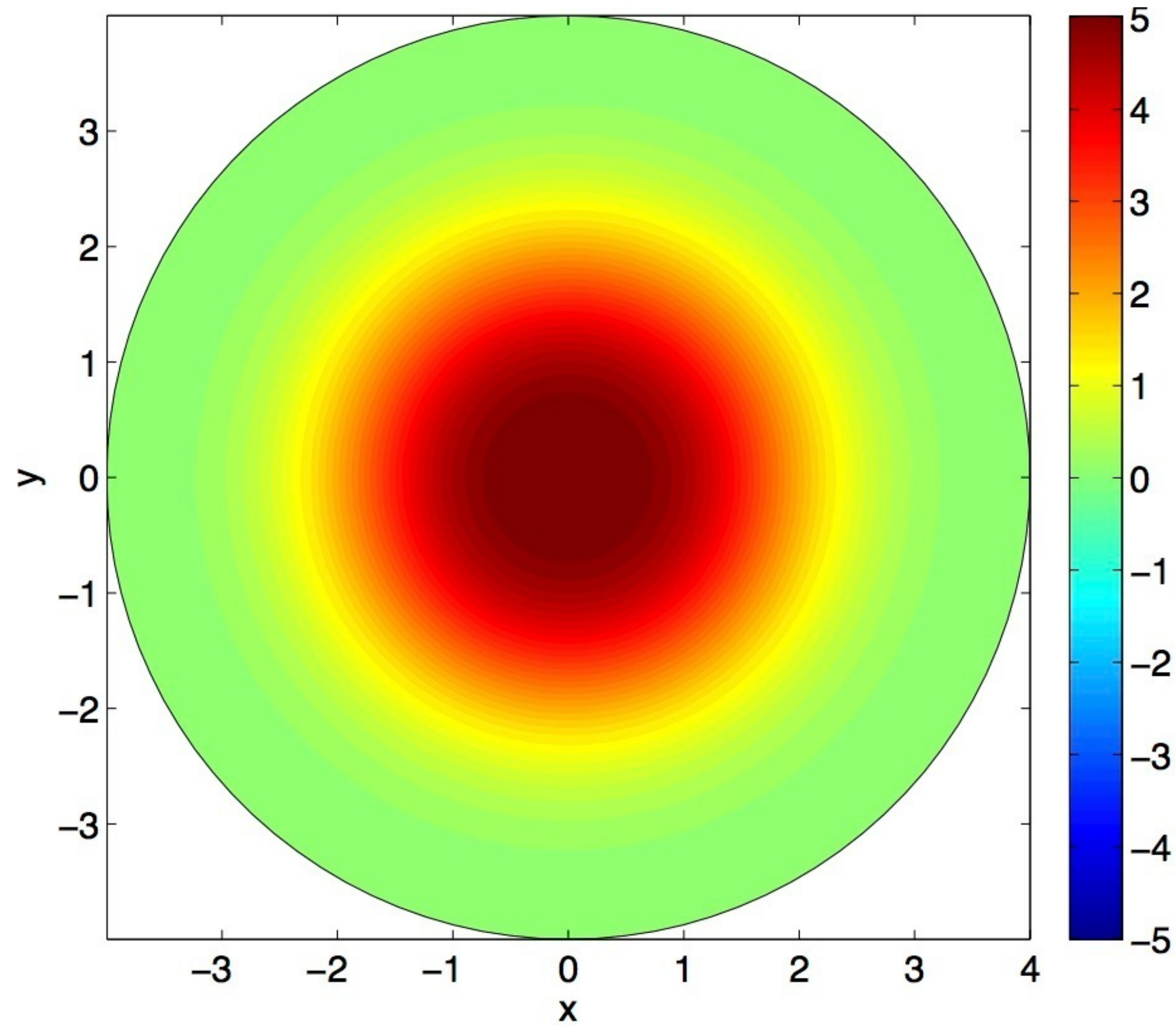
Completeness Theorem

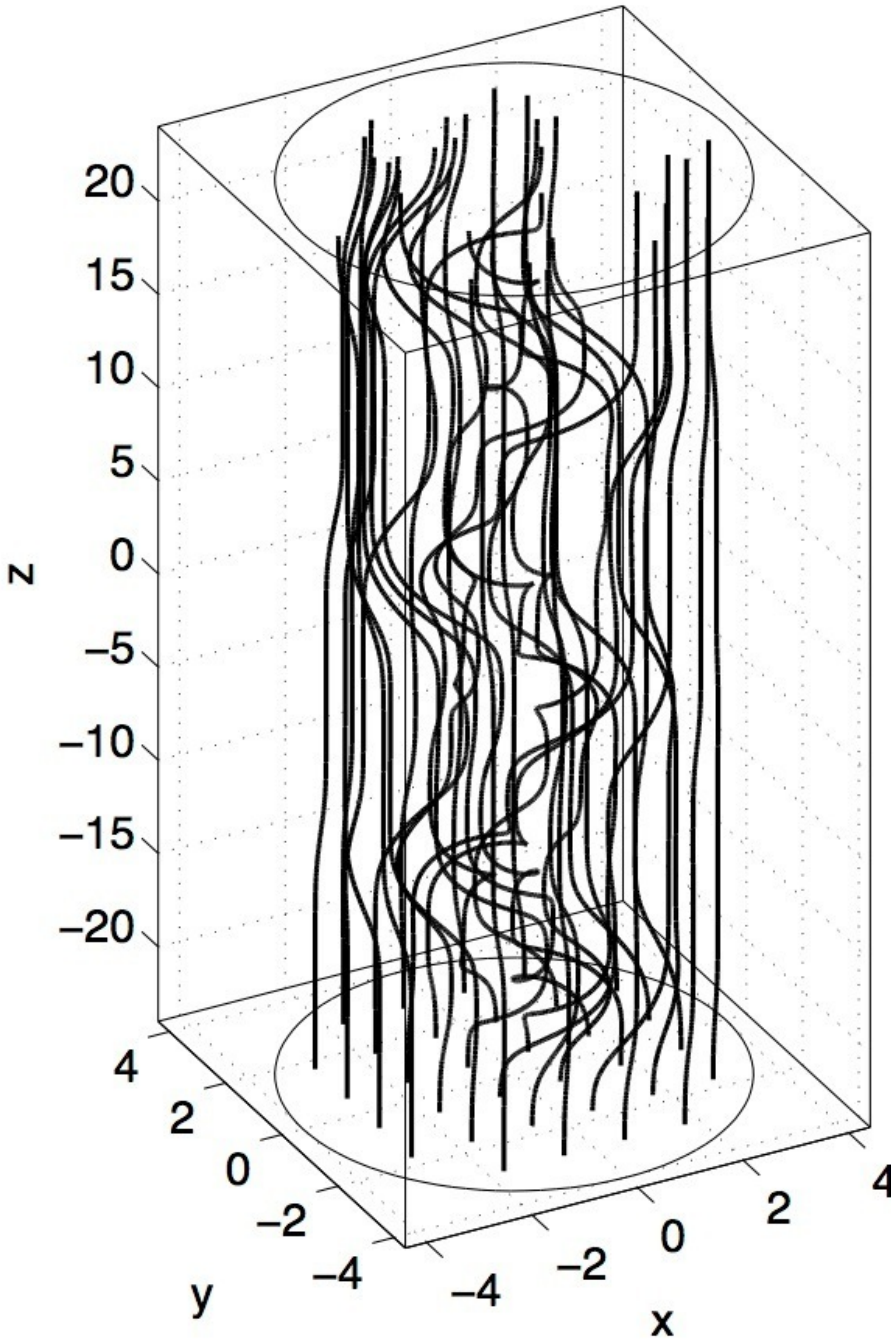
The flux function is a **necessary and sufficient condition** for two flux ropes to have the same field line mapping.

Yeates & Hornig, *Phys. Plasmas* (2013)

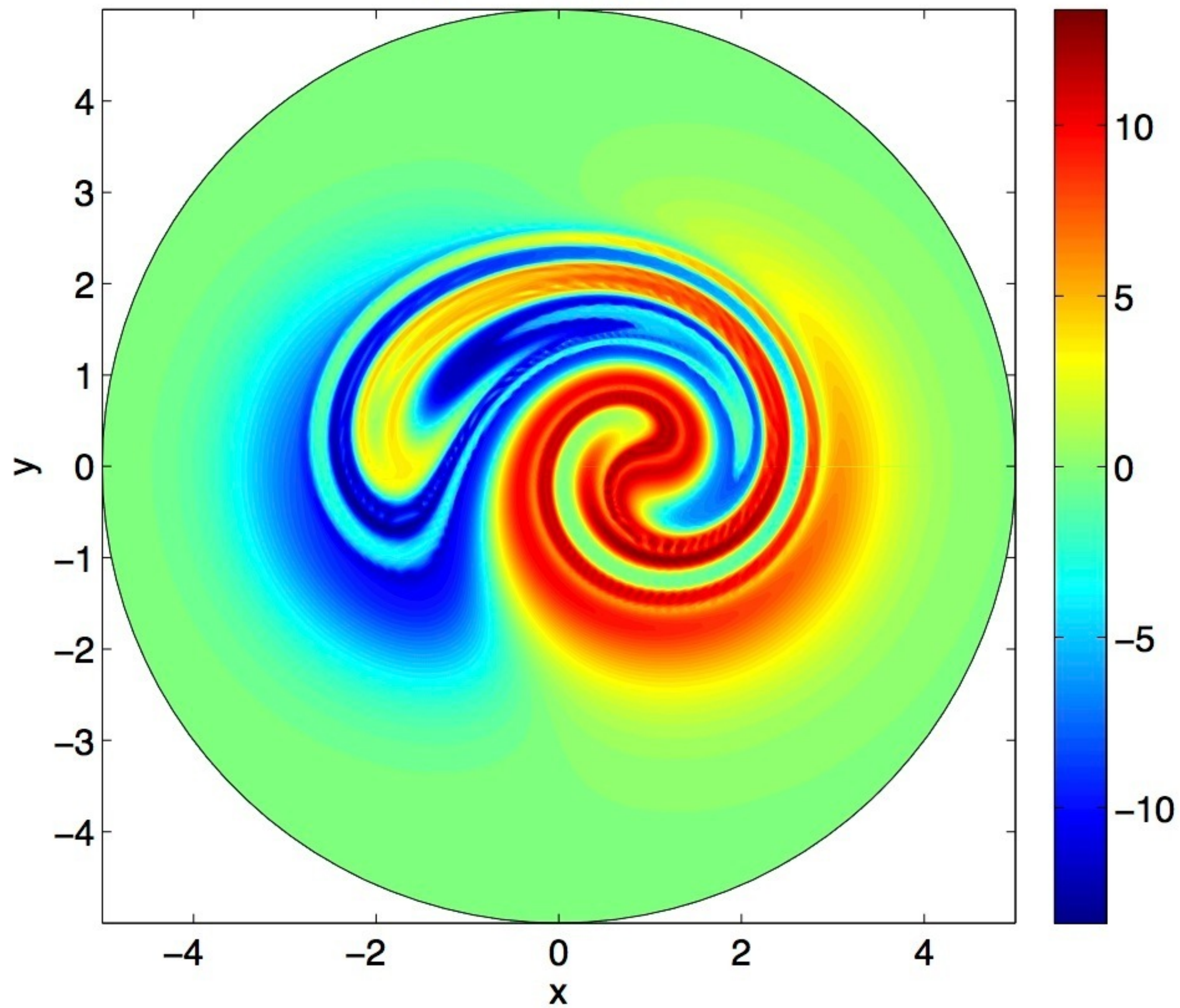


Flux function:





Flux function:

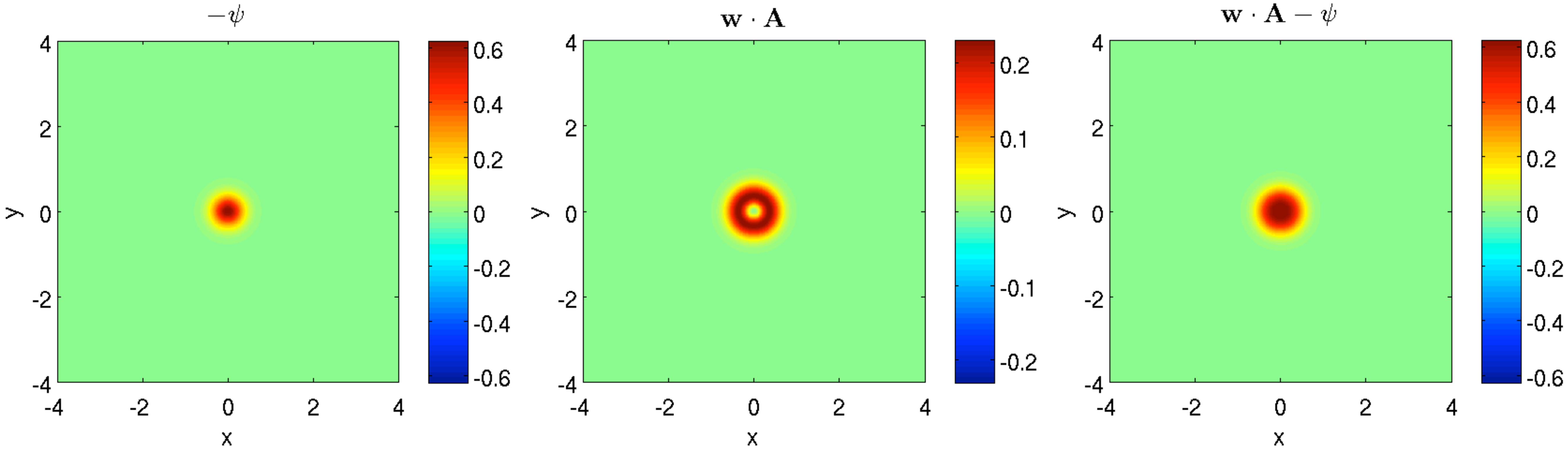


Time evolution

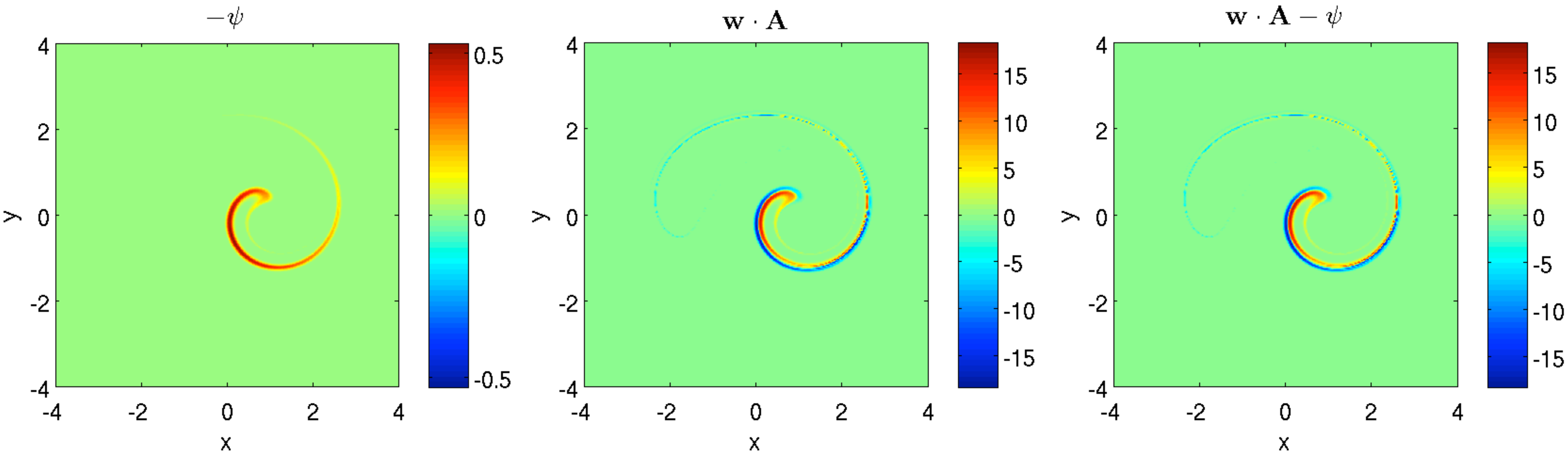
$$\frac{\partial \mathcal{A}}{\partial t}(\mathbf{x}) = \Big(-\psi + \mathbf{w}\cdot\mathbf{A} \Big)_{F_h(\mathbf{x})}$$

- Recovers single-site reconnection rate where $\mathbf{w} = 0$.
- But contains **all** information about how the global topology is changing.

Single site:



Now add a background field:



Topological effectiveness of a reconnection site depends on the background field structure.

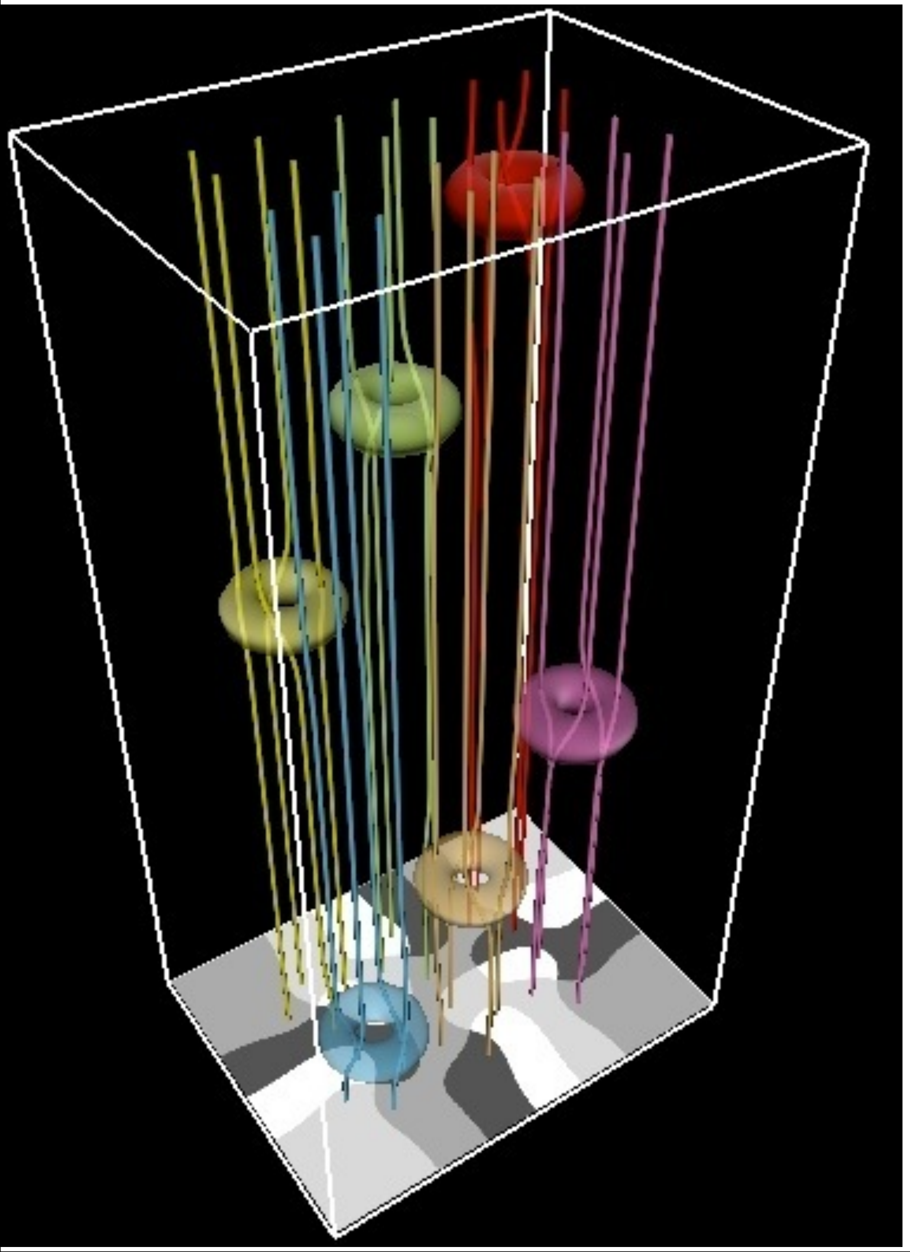
A global reconnection rate?

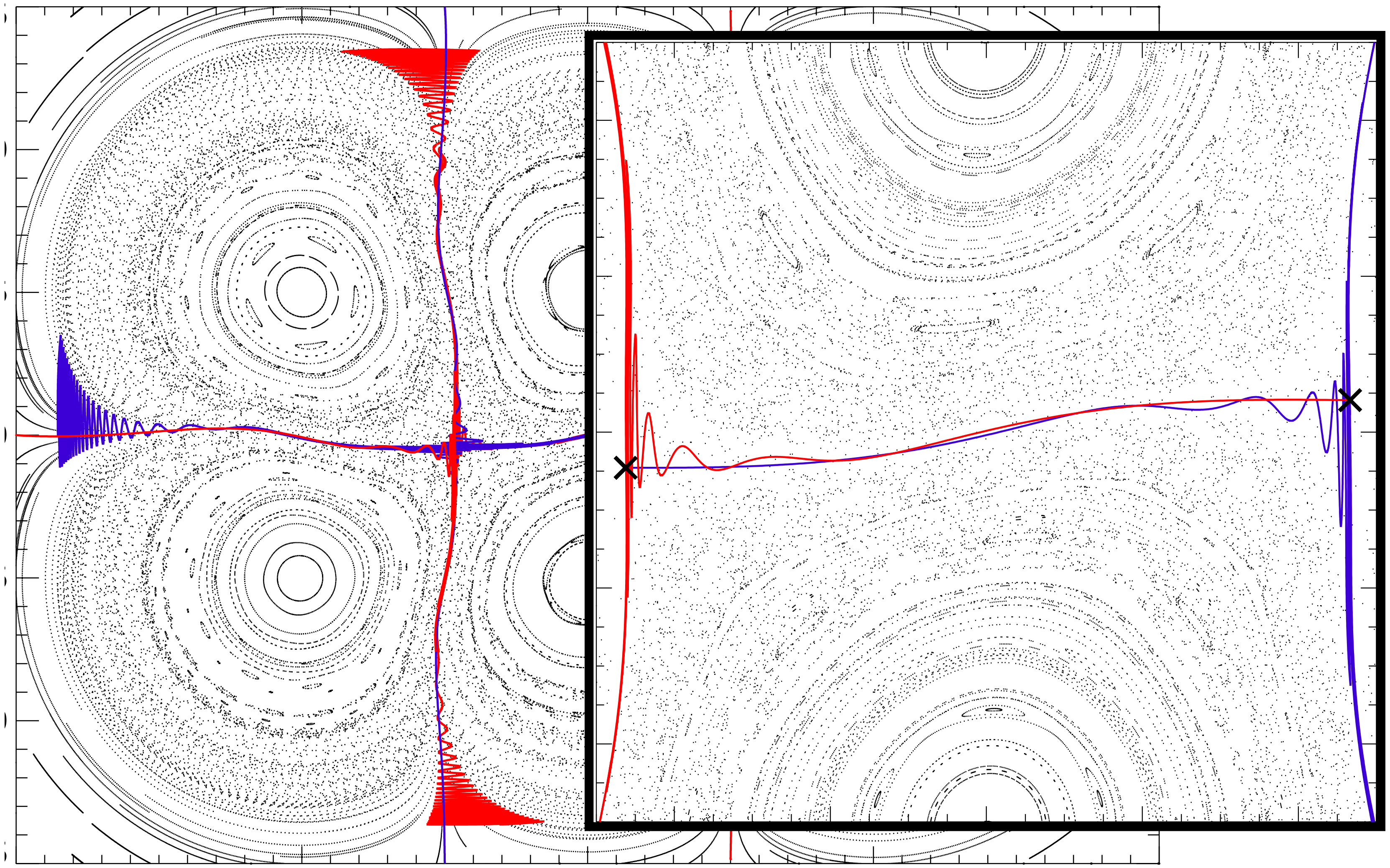
Idea: Identify a discrete set of "topological" field lines.

1. Points where $\mathbf{w} = 0$. (instantaneously un-reconnecting lines)

2. Points where $F_h(\mathbf{x}) = \mathbf{x}$. (fixed points of the field line mapping)

The latter **partition** the flux but chaotic field lines lead to leakage of flux (partial barriers).





Yeates & Hornig, *Phys. Plasmas* (2011)

Summary

- Reconnection occurs in flux ropes even though there are no X-points.
- It takes place **locally** but its effect should be measured **globally**.
- A **flux function** captures the global field structure, and its rate of change may be related to reconnection events.

Ongoing work:

- How are different types of evolution characterized in the flux function?
- Applying these ideas to the LAPD experiments.

References:

Yeates & Hornig, *Phys. Plasmas* (2011, 2013)
Yeates & Hornig (arxiv.org/abs/1304.8064)
Prior & Yeates (submitted)

Science & Technology Facilities Council

http://www.maths.dur.ac.uk/~bmjg46/

Durham University