Impact of Magnetic Topology on Plasma Dynamics

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with

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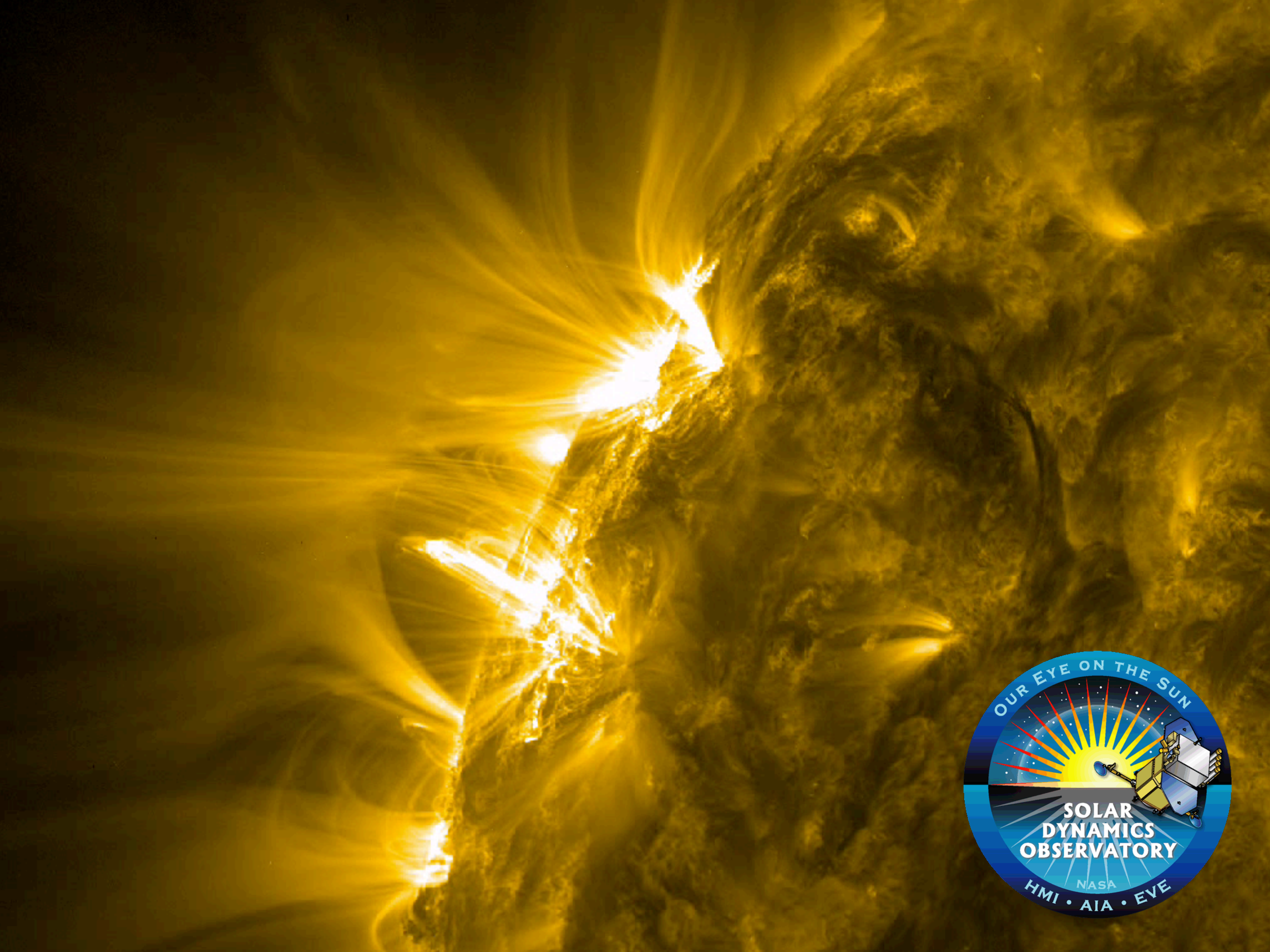
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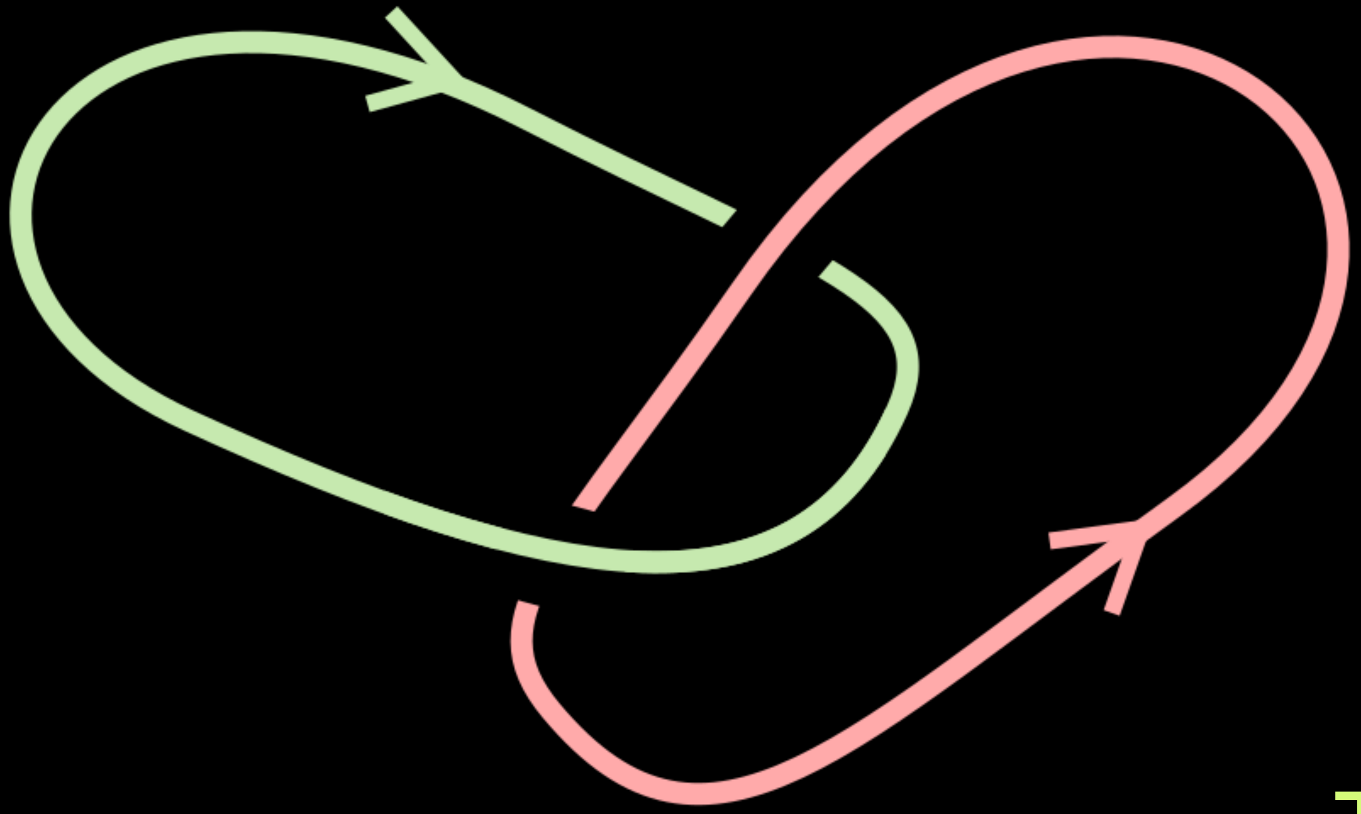
Ideal MagnetoHydroDynamics

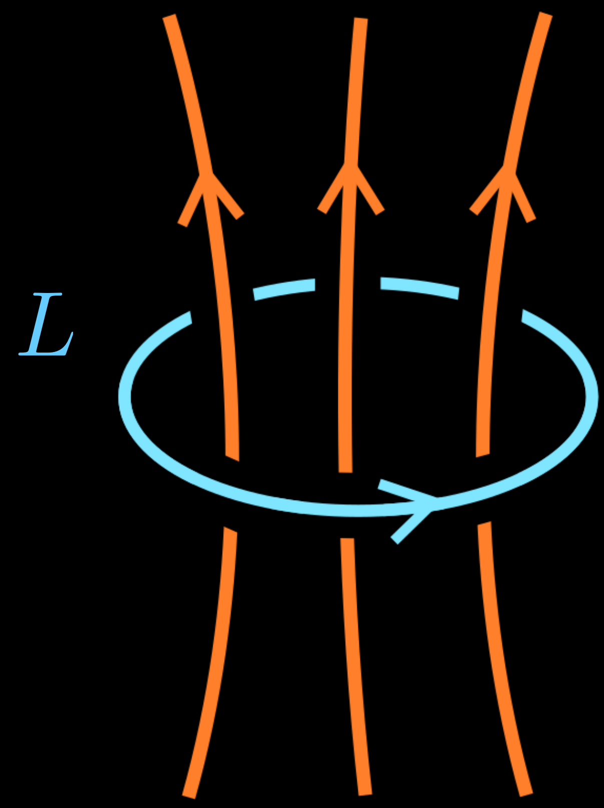
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$



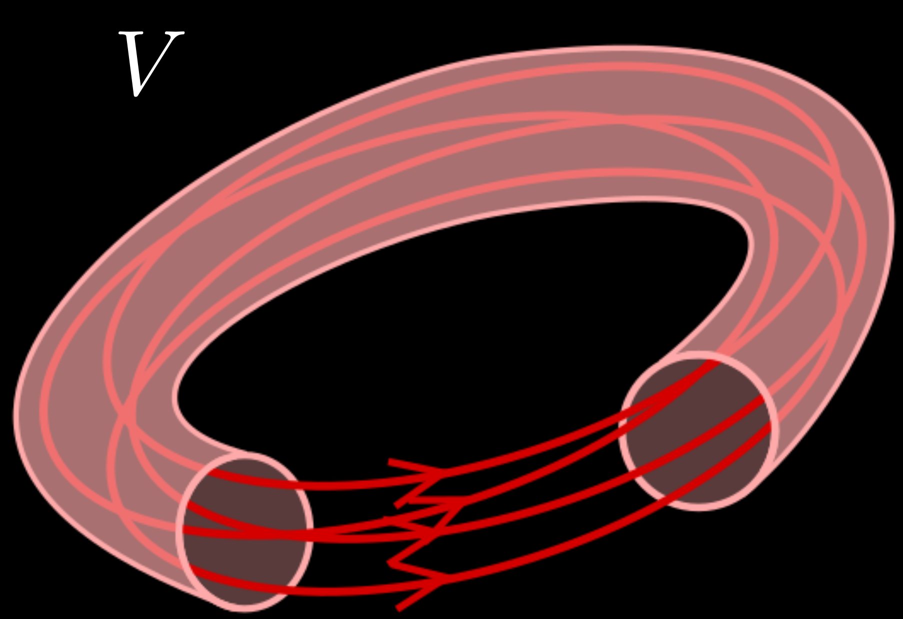
Topology of magnetic field lines is frozen-in.

Ideal MHD invariants



Field line helicity: the magnetic flux through a closed field line:

$$\mathcal{A}(L) = \oint_L \mathbf{A} \cdot \mathrm{dl} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

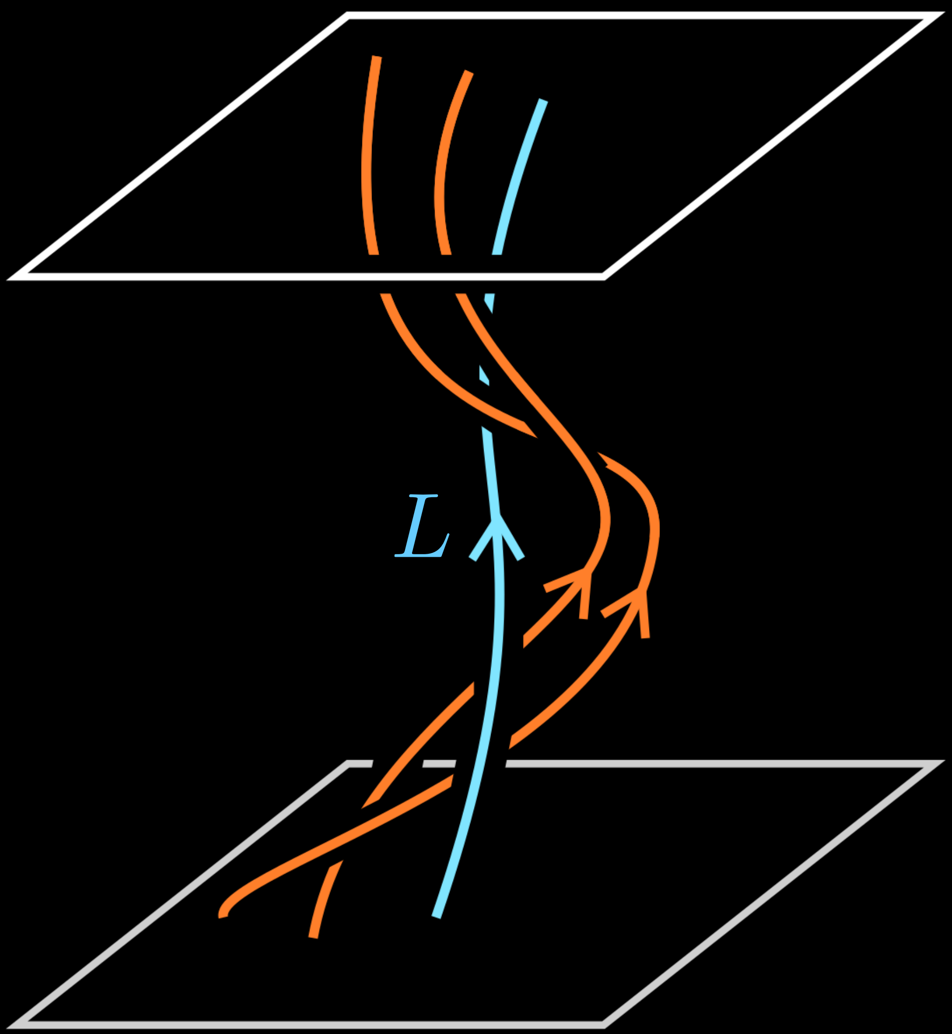


Magnetic helicity: integral over a magnetic subdomain:

$$H(V) = \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V$$

Ideal MHD invariants

Can be extended to open magnetic fields if line-tied and gauge fixed.



$$\mathcal{A}(L) = \int_L \mathbf{A} \cdot \mathrm{d}\mathbf{l}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Real plasmas: dissipation is inevitable!

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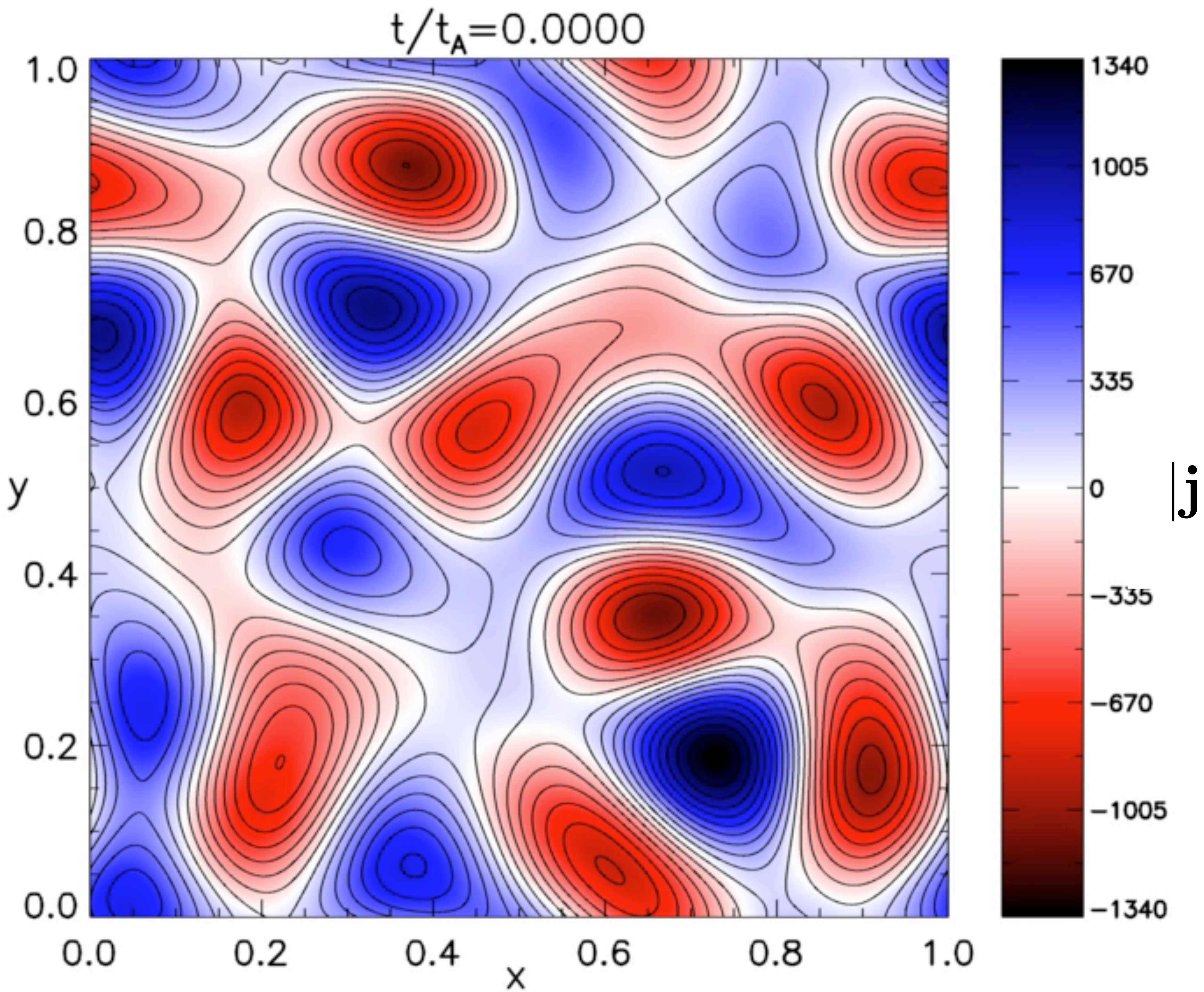
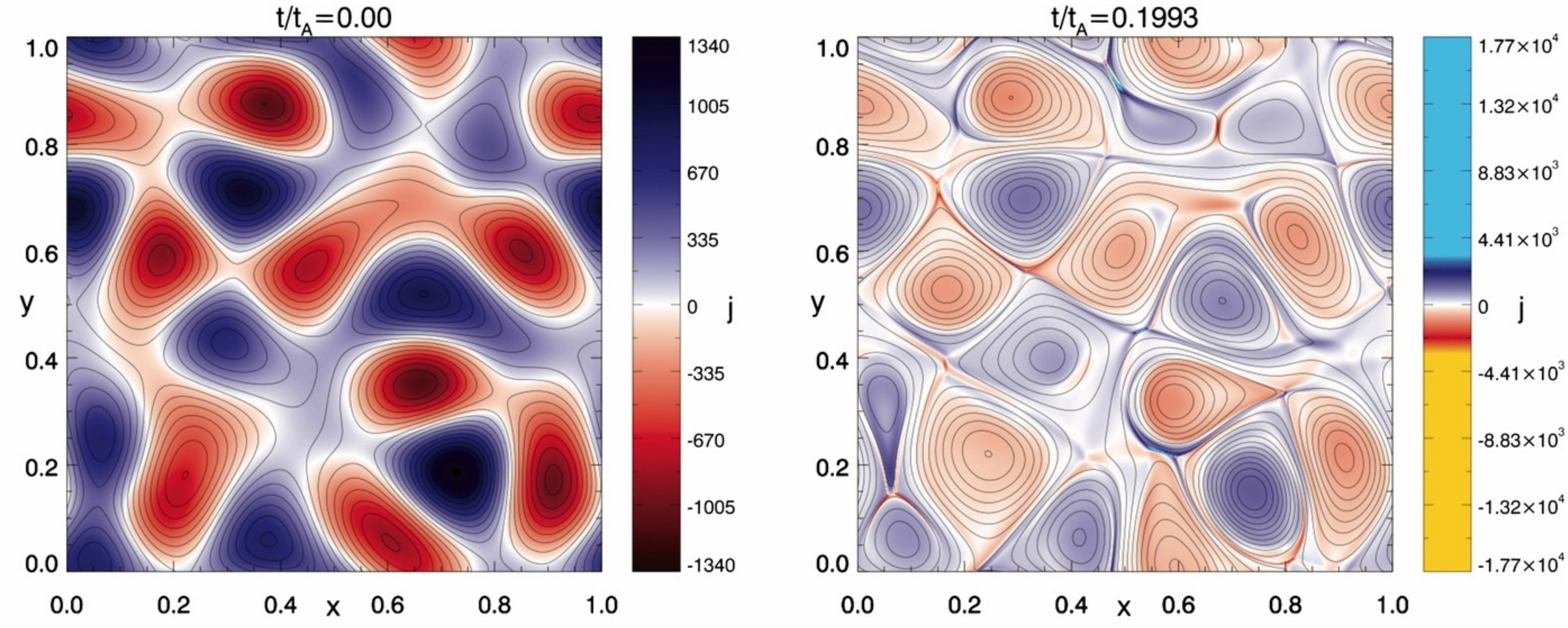


e.g. Rappazzo & Parker., *ApJL* (2013)

Real plasmas: dissipation is inevitable!



e.g. Rappazzo & Parker., *ApJL* (2013)

Resistive MHD

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}$$

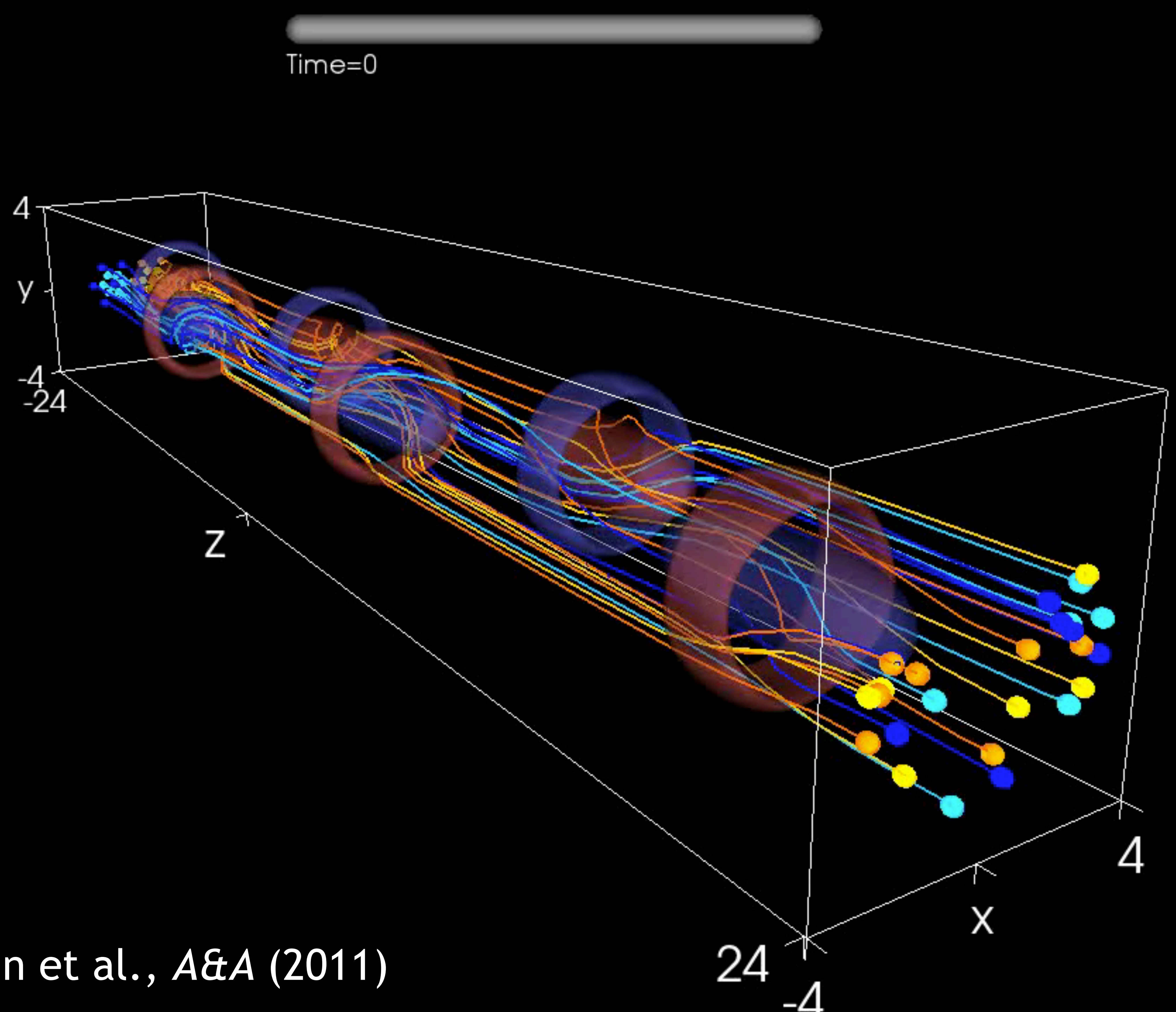
$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

Field line helicities change, but total helicity is approximately invariant.

Berger, *GAFD* (1984)

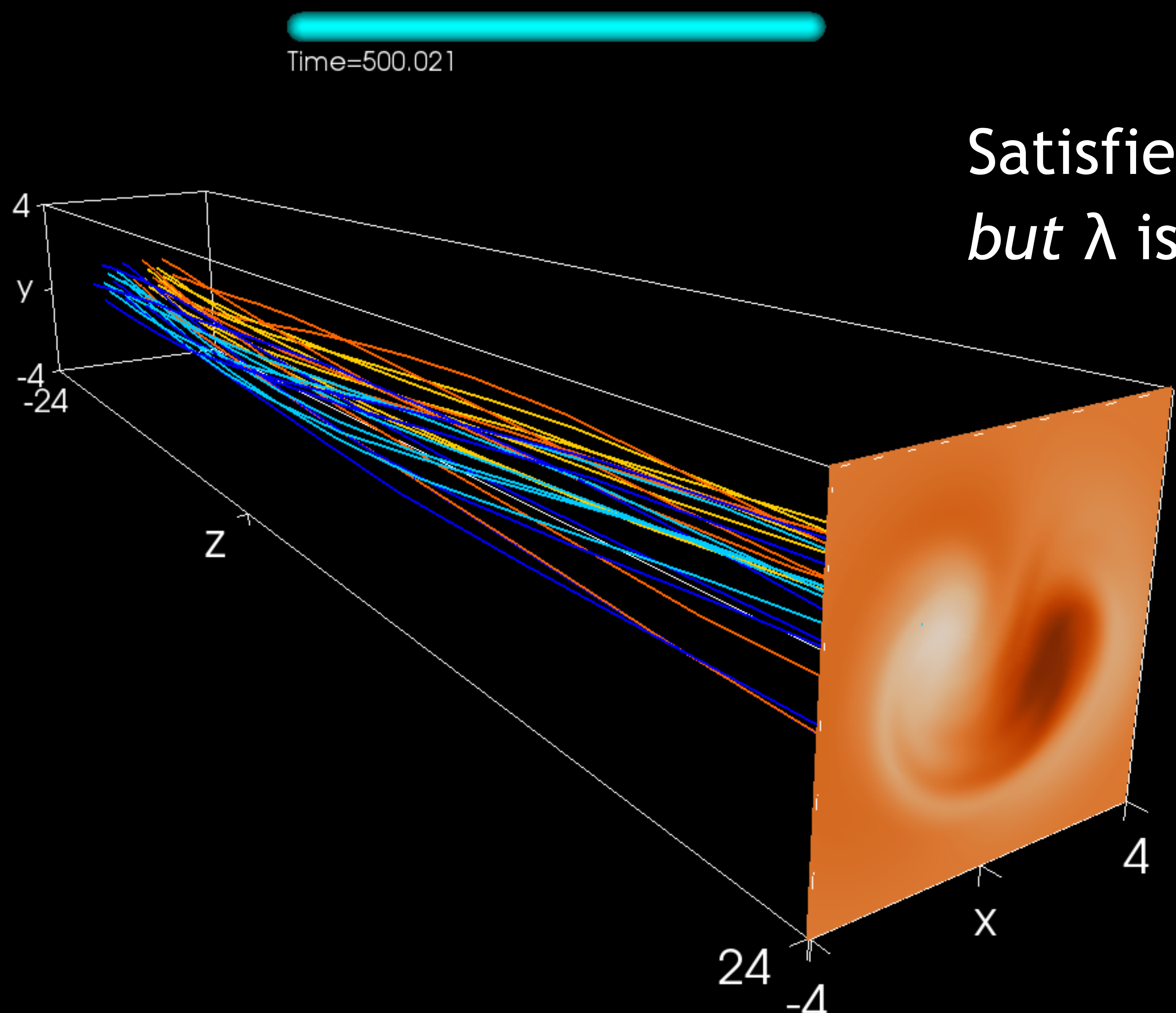
$$\frac{\mathrm{d}}{\mathrm{d}t} H(V) = -2\eta \int_V \mathbf{j} \cdot \mathbf{B} \, \mathrm{d}V$$

Dundee braiding simulation



Pontin et al., *A&A* (2011)

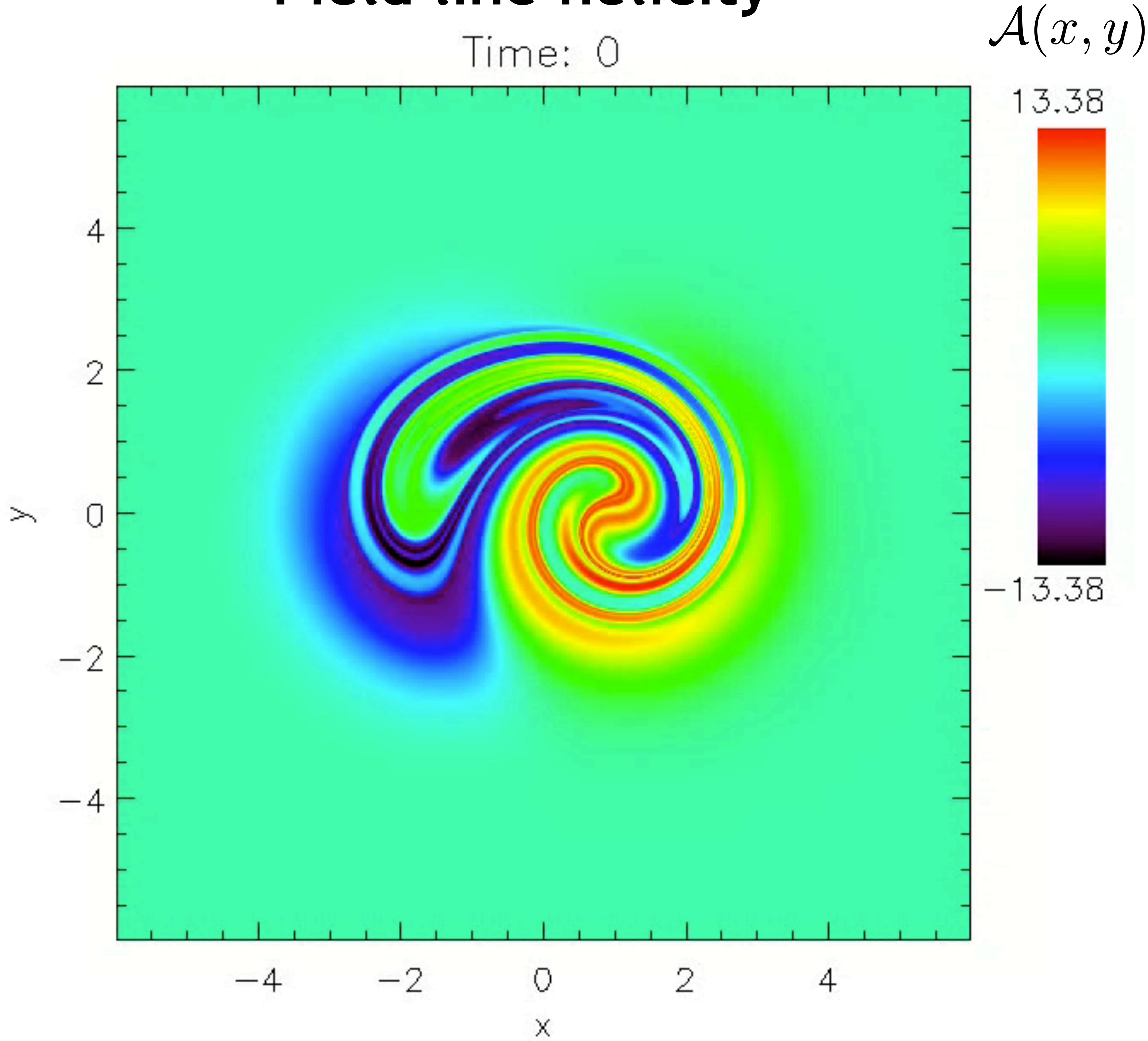
The final state



Satisfies $\nabla \times \mathbf{B} = \lambda \mathbf{B}$
but λ is non-uniform.

<u>cf. Taylor hypothesis</u>: *H* is only constraint ⇒ λ uniform.

Field line helicity



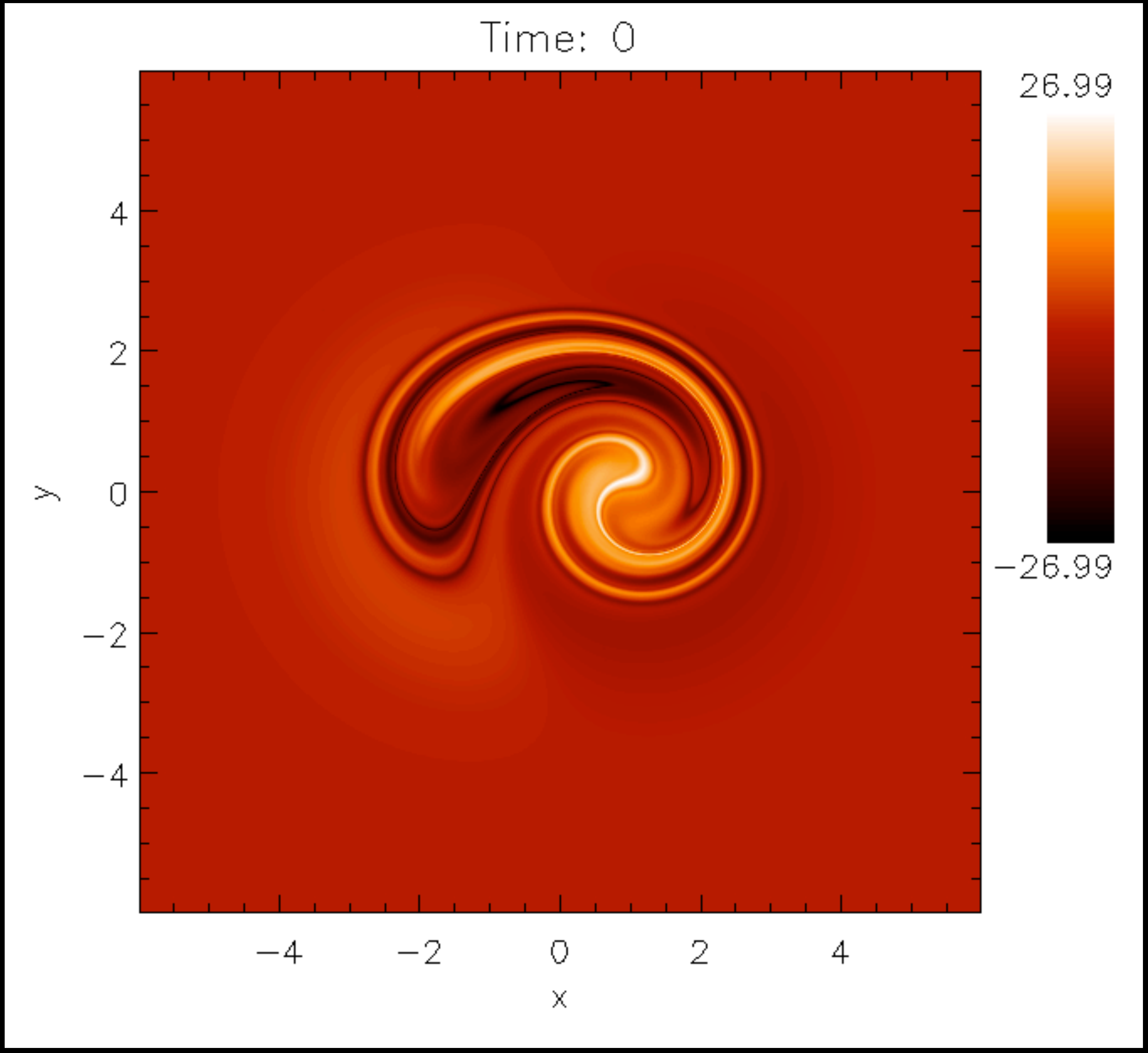
Evolution of field line helicity

$$\frac{\partial \mathcal{A}}{\partial t} + \mathbf{w} \cdot \nabla \mathcal{A} = \mathbf{w} \cdot \mathbf{A} - \psi$$

Volume dissipation

$$\psi = \eta \int_L \mathbf{j} \cdot \mathrm{d}\mathbf{l}$$

(same as for *H*)

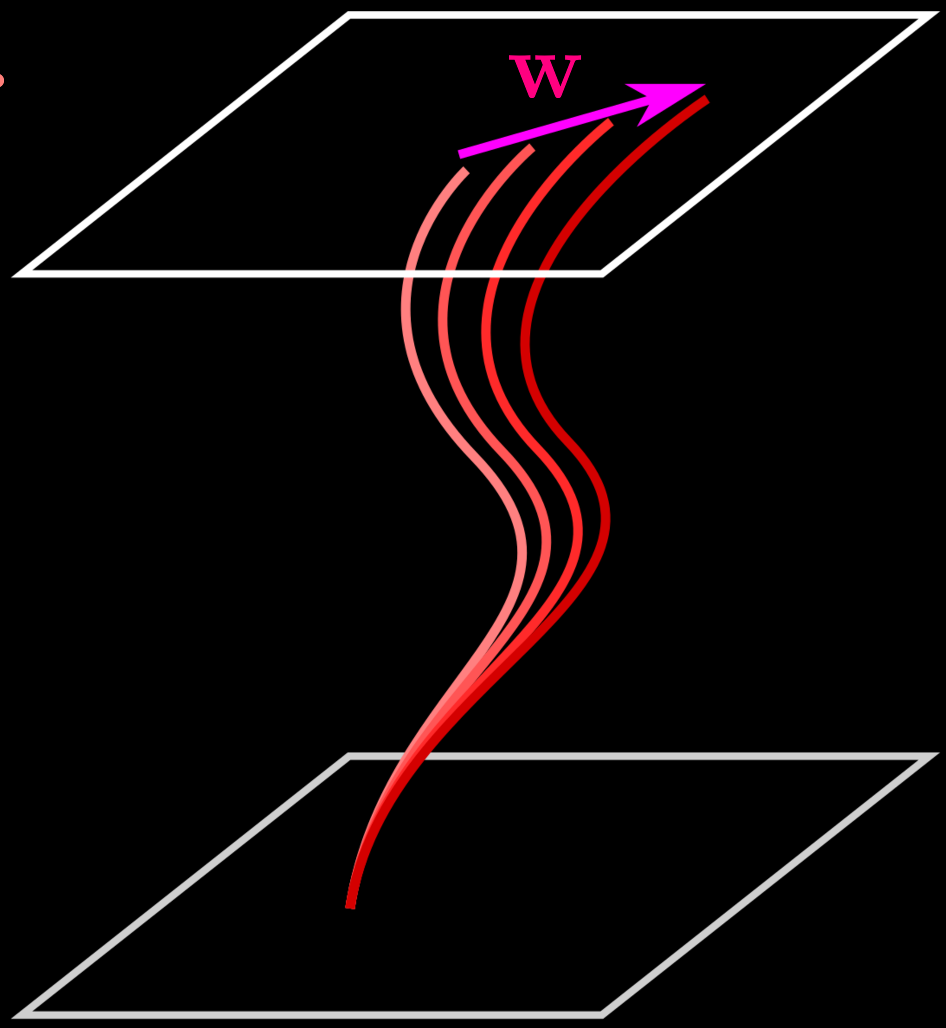


Evolution of field line helicity

$$\frac{\partial \mathcal{A}}{\partial t} + \mathbf{w} \cdot \nabla \mathcal{A} = \boxed{\mathbf{w}} \cdot \mathbf{A} - \psi$$

Velocity of field line end-points.

$$\mathbf{w} = \frac{\left(\mathbf{E} - \nabla \psi\right) \times \mathbf{B}}{|\mathbf{B}|^2}$$



Evolution of field line helicity

$$\frac{\partial \mathcal{A}}{\partial t} + \mathbf{w} \cdot \nabla \mathcal{A} = \boxed{\mathbf{w} \cdot \mathbf{A}} - \psi$$

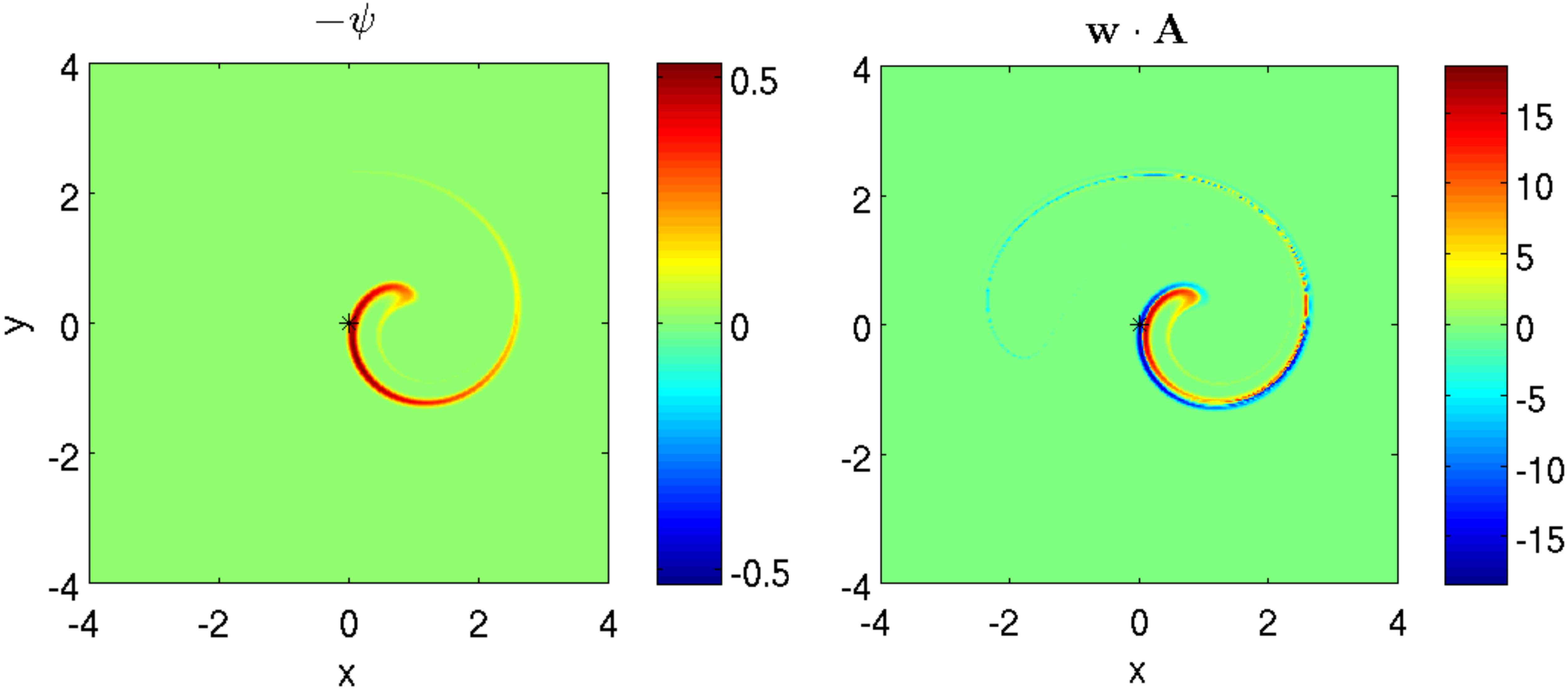
$$\sim \left(\frac{L}{l}\right) \psi$$

Term depends on chosen gauge.

L = lengthscale of local variations in **B**

l = lengthscale of field line mapping

Typical behaviour



Evolution of field line helicity

$$\frac{\partial \mathcal{A}}{\partial t} + \boxed{\mathbf{w} \cdot \nabla \mathcal{A}} = \mathbf{w} \cdot \mathbf{A} - \psi$$

$$\sim \left(\frac{L}{l}\right)^2 \psi$$

Advection with motion of the field line end-point.

Dominant term when mapping gradients large enough!

Summary

Relaxation of astrophysical plasmas is constrained by magnetic topology.

Realistic magnetic fields have small length scales in field-line integrated quantities.

Improved relaxation hypothesis:
Reconnection is efficient at redistributing field line helicity but not at destroying it.

Russell, Yeates, Hornig & Wilmot-Smith, *Phys. Plasmas* **22**, 032106 (2015)