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Does a potential magnetic field contain helicity?

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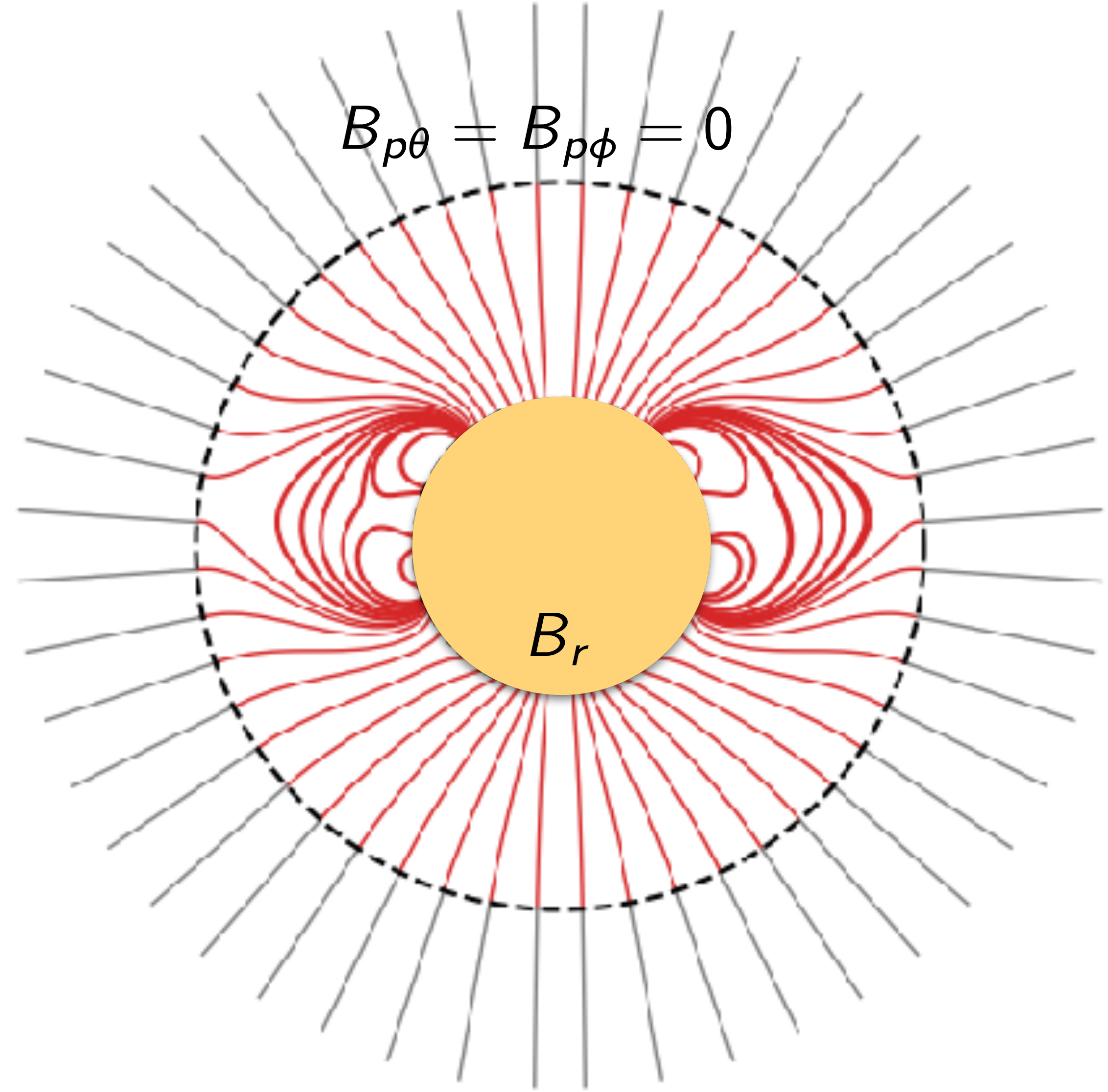
Outline

- Potential magnetic field (no volume currents):

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \qquad \boldsymbol{B}_p = \nabla \phi$$
$$\nabla \times \boldsymbol{B}_p = \boldsymbol{0} \quad \Longrightarrow \quad \nabla^2 \phi = 0$$

- For the solar corona, we impose a “source surface” outer boundary to model the streamer structure.
  [Altschuler-Newkirk *Solar Phys,* 1969]
  [Schatten et al *Solar Phys*, 1969]



- **Theory: how can a potential field contain helicity?**

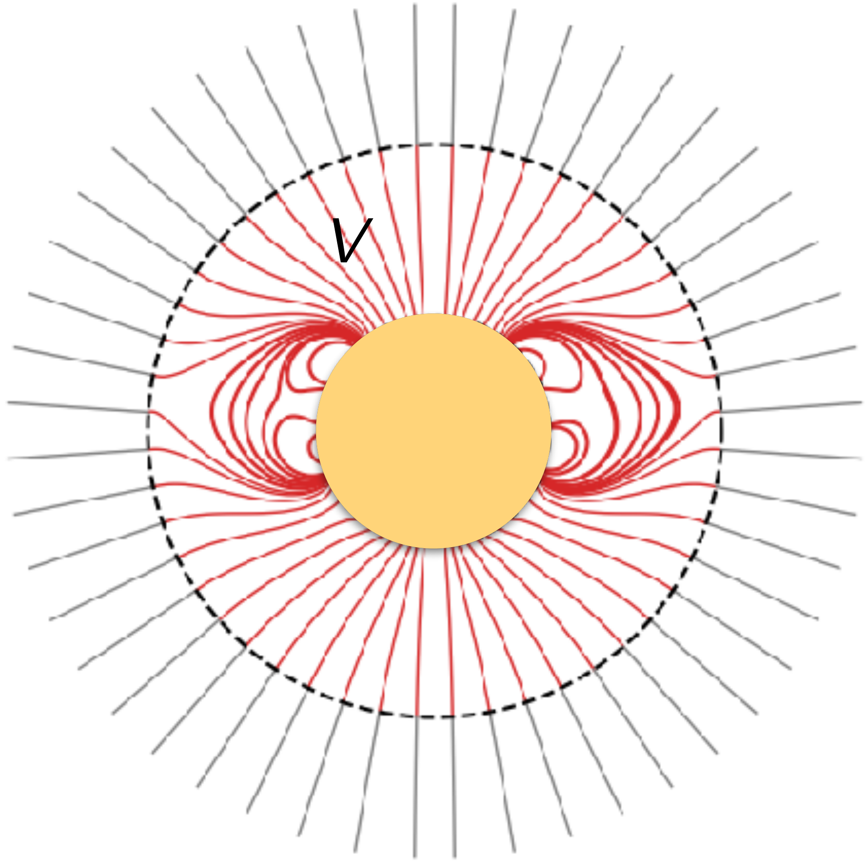
- **Computations: how much helicity would a potential solar corona contain?**

Theory

How can a potential field contain helicity?

Helicity of a potential field

$$H = \int_V \boldsymbol{A}_p \cdot \boldsymbol{B}_p \, \mathrm{d}V \qquad \text{where} \qquad \boldsymbol{B}_p = \nabla \times \boldsymbol{A}_p$$



- By choosing $\boldsymbol{A}_p$ we can give H any arbitrary value:

$$\boldsymbol{A}_p \to \boldsymbol{A}_p + \nabla\chi \qquad H \to H + \oint_{\partial V} \chi \boldsymbol{B}_p \cdot \boldsymbol{n} \, \mathrm{d}S$$

- Most logical choice is to make it vanish by choosing

$$A_{pr} = 0 \qquad \nabla \cdot \boldsymbol{A}_p = 0$$

so

$$H(V) = \int_V \boldsymbol{A}_p \cdot \nabla\phi \, \mathrm{d}V = \oint_{\partial V} \phi \boldsymbol{A}_p \cdot \boldsymbol{dS} - \int_V \phi \nabla \cdot \boldsymbol{A}_p \, \mathrm{d}V = 0$$

[eg. Berger *A&A*, 1988]

- **Main observation:** if we subdivide

$$V = V_1 + V_2 + \ldots + V_n$$

then the individual $H(V_i)$ will be non-zero in general…

Our vector potential $A_{pr} = 0 \qquad \nabla \cdot \boldsymbol{A}_p = 0$

- We can write

$$\boldsymbol{A}_p = \nabla \times \left(P\hat{\boldsymbol{r}}\right) \quad \Longrightarrow \quad \nabla_h^2 P = -B_{pr} \quad \text{on each spherical surface}$$

- This gauge minimises $\int_V |\boldsymbol{A}_p|^2 \, \mathrm{d}V$ because

$$\int_V |\boldsymbol{A}_p + \nabla\chi|^2 \, \mathrm{d}V = \int_V |\boldsymbol{A}_p|^2 \, \mathrm{d}V + \int_V |\nabla\chi|^2 \, \mathrm{d}V + 2\oint_{\partial V} \chi \boldsymbol{A}_p \cdot \boldsymbol{dS} - 2\int_V \chi\nabla \cdot \boldsymbol{A}_p \, \mathrm{d}V$$

[Gubarev et al. *PRL*, 2001]

[cf. Yeates-Page *J Plasma Phys*, 2018]

- It is the "potential field limit" of the more general **poloidal-toroidal** vector potential

$$\boldsymbol{A}^{PT} = T\hat{\boldsymbol{r}} + \nabla \times \left(P\hat{\boldsymbol{r}}\right) \qquad \nabla_h^2 T = -J_r$$

for which H is the Berger-Field relative helicity (with potential reference):

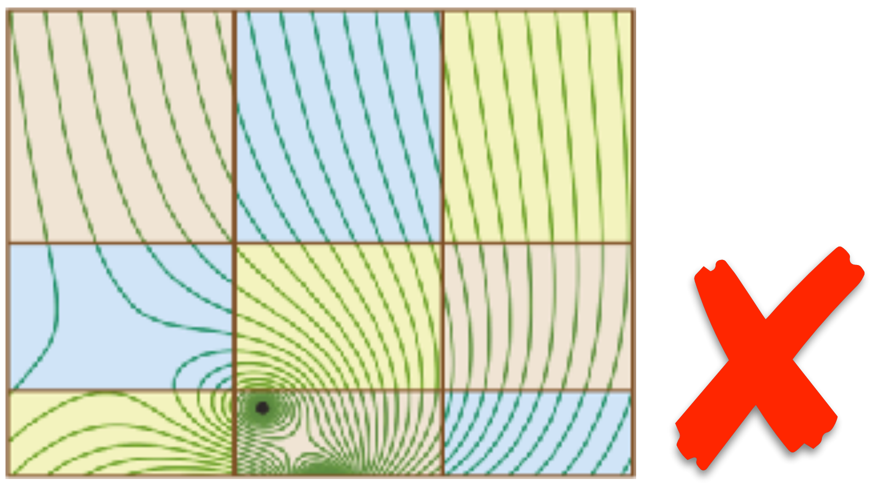
$$H_r(V) = \int_V \boldsymbol{A}^{PT} \cdot \boldsymbol{B}^{PT} \, \mathrm{d}V$$

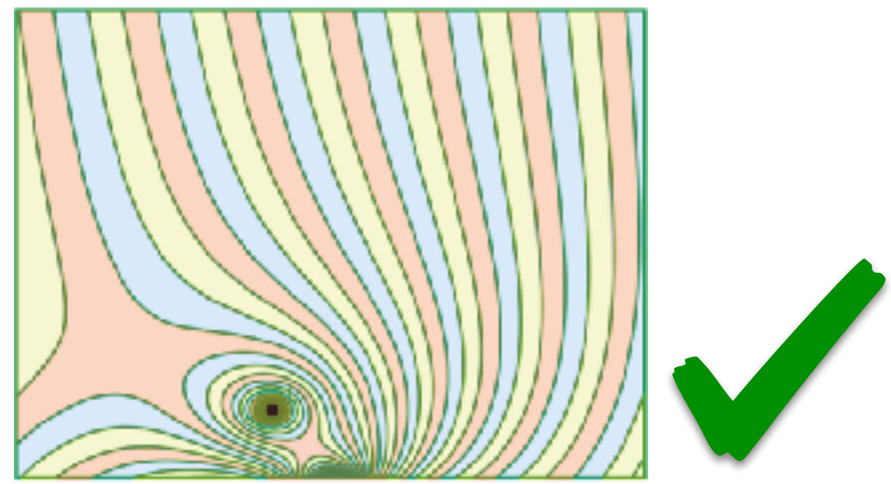
[cf. Berger-Hornig *J Phys A*, 2018]

In general the Berger-Field relative helicity is $H_r(V) = \int_V \left(\boldsymbol{A} + \boldsymbol{A}_P\right) \cdot \left(\boldsymbol{B} - \boldsymbol{B}_P\right) \mathrm{d}V$

Field line helicity

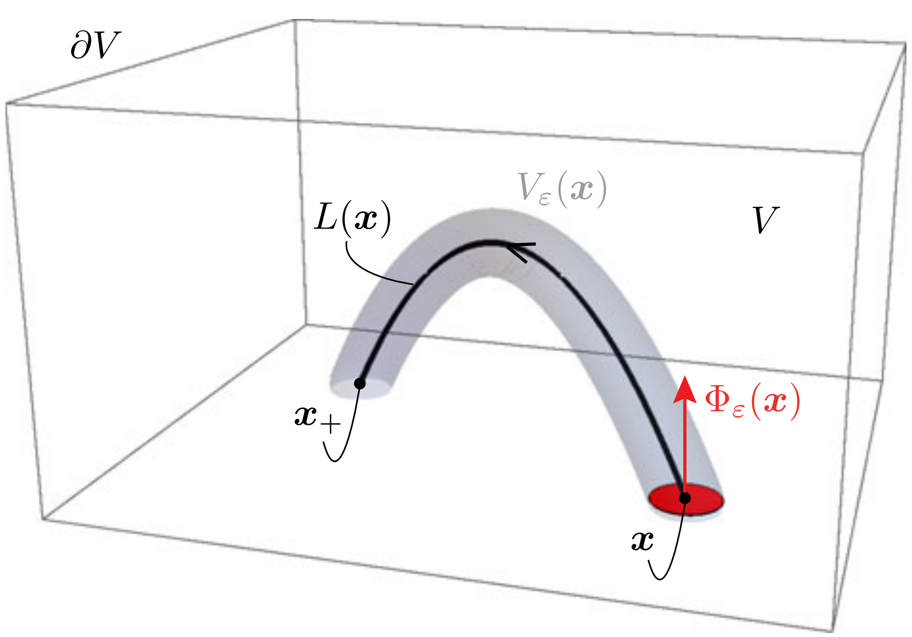
- For physical relevance we should subdivide V into **magnetic subdomains**:





Then each $h(V_i)$ will be an ideal invariant [for line-tied boundaries].

- Taking a limiting domain around every field line gives the **field line helicity**:



[Yeates-Page *J Plasma Phys*, 2018]

$$\mathcal{A}(L) = \lim_{\epsilon\to 0} \frac{\int_{V_\epsilon(L)} \boldsymbol{A}_p \cdot \boldsymbol{B}_p \mathrm{d}V}{\Phi(V_\epsilon(L))} = \int_L \boldsymbol{A}_p \cdot \mathrm{d}\mathbf{l}$$

[Berger *A&A*, 1988; Yeates-Hornig *Phys Plasmas* 2013; Aly *Fluid Dyn Res* 2018]

- This is an ideal invariant "density" of helicity:

$$\int_{\{L\}} \mathcal{A}(L)\,\mathrm{d}\Phi = H(V)$$

- For any field with no closed loops, we can write this as a boundary integral

$$H(V) = \frac{1}{2}\int_{\partial V} \mathcal{A}|B_{pr}|\,\mathrm{d}S$$

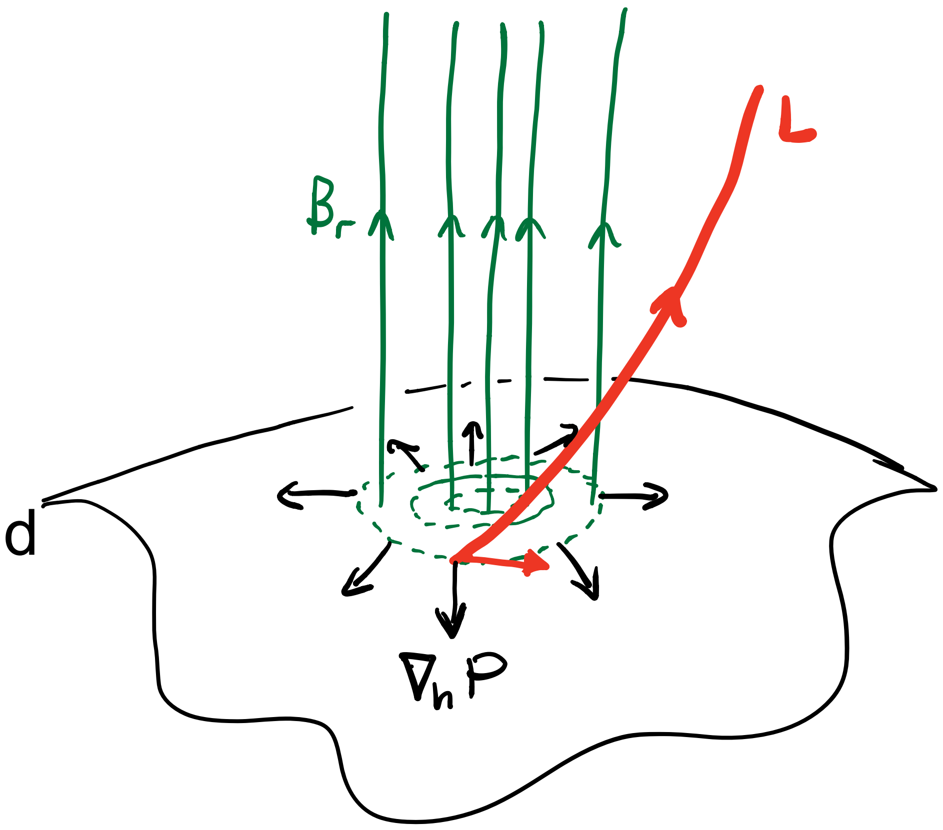
Physical meaning

- In our gauge, for a potential field,

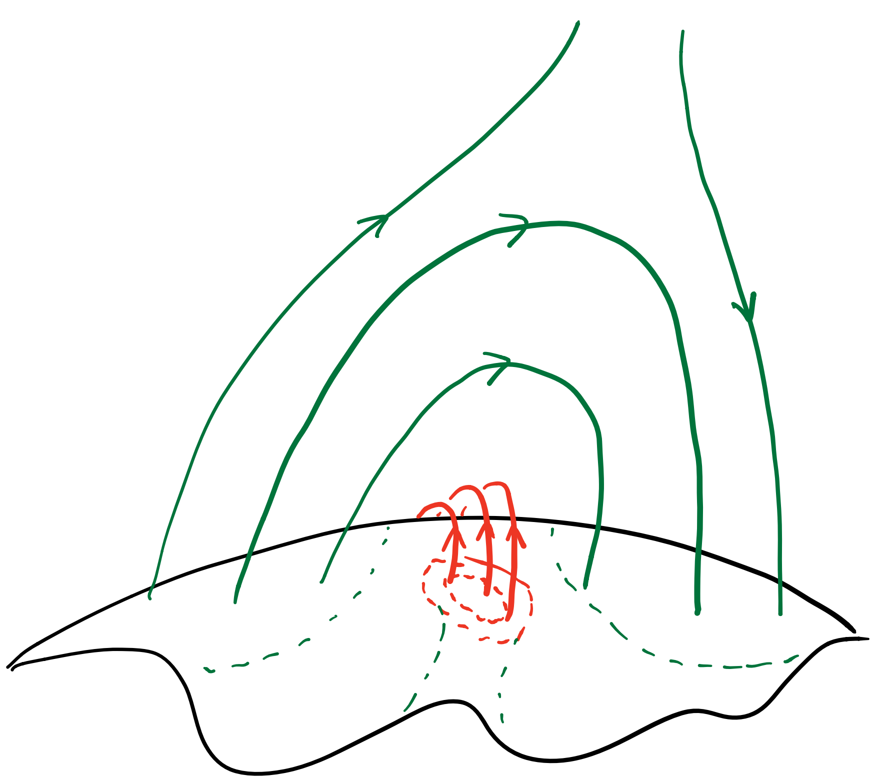
$$\mathcal{A}(L) = \int_L \boldsymbol{A}_p \cdot d\boldsymbol{l} = \int_L \hat{\boldsymbol{r}} \cdot \left(d\boldsymbol{l} \times \nabla_h P\right)$$

so (potential field) FLH measures "winding around concentrations of B_{pr}".

[cf. Prior-Yeates *ApJ*, 2014]



- Even a potential field can contain linking like this in 3D:



For an arched field line you can interpret FLH as the magnetic flux underneath.

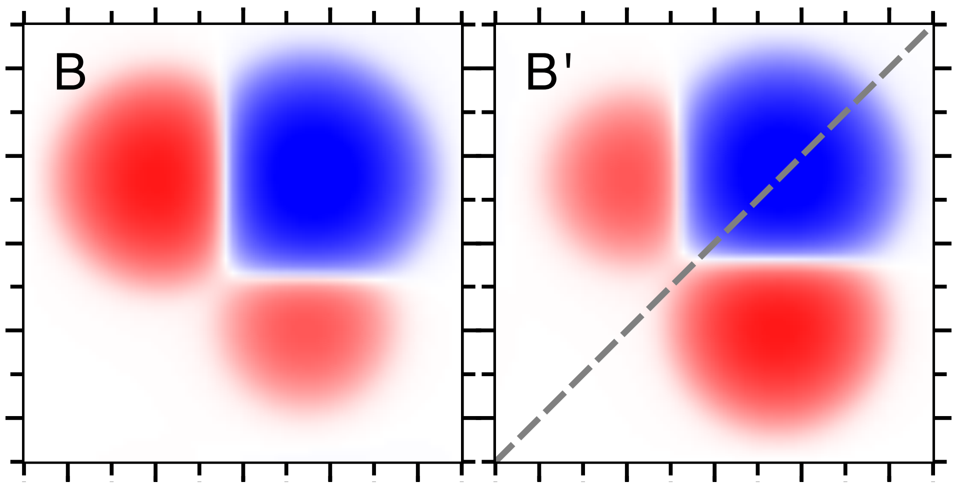
[Yeates-Hornig *A&A*, 2016]
[cf. Antiochos *ApJ*, 1987]

Minimal helicity content

- Since the potential field is a minimum-energy state, I think of the FLH distribution in our “minimal gauge” as the **minimum helicity state**.

- Since a potential field is determined entirely by B_r on the solar surface, the minimal helicity is really a consequence of that pattern.

[cf. Bourdin-Brandenburg *ApJ*, 2018]



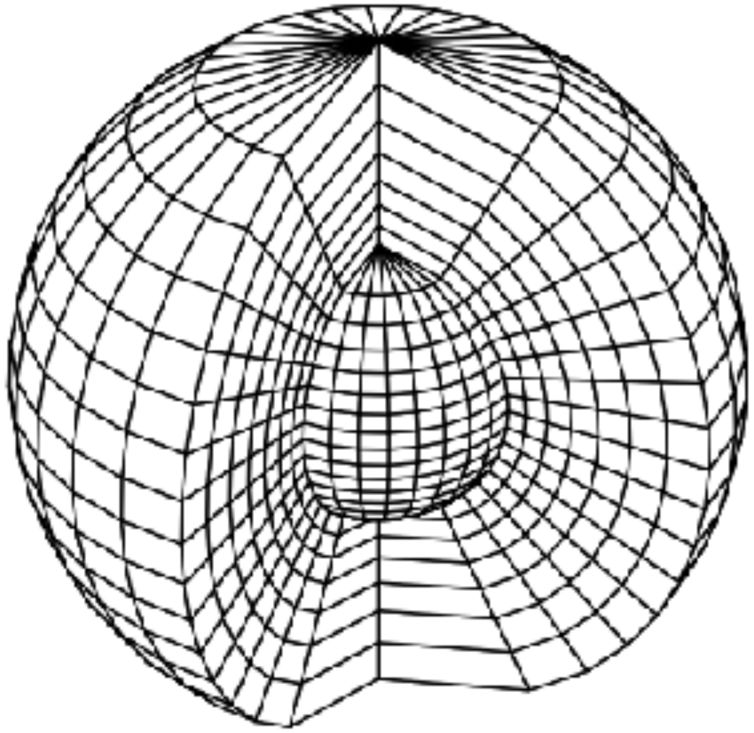
- In future slides I will measure this minimal helicity content with the (non-ideal-invariant) total **unsigned helicity**

$$\overline{H}(V) = \frac{1}{2}\int_{\partial V} |\mathcal{A}B_{pr}| \, \mathrm{d}S$$

Computations

How much helicity would a potential solar corona contain?

Numerical methods



- Regular grid 60 x 180 x 360 in $(\log(r/r_0), \cos\theta, \phi)$

- Finite-difference PFSS code in Python: https://github.com/antyeates1983/pfss
  [cf. van Ballegooijen-Priest-Mackay *ApJ*, 2000]

- Compute vector potential using [cf. Amari et al 2013; Moraitis et al. 2018]

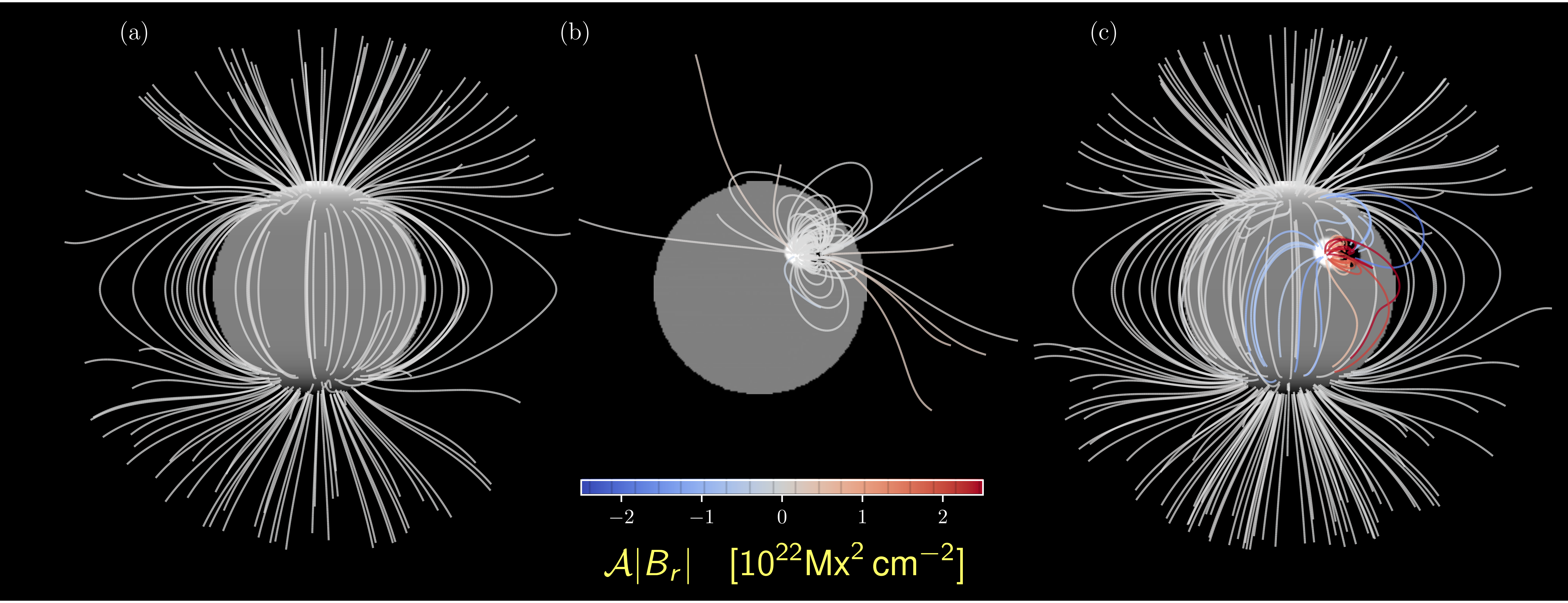
$$\boldsymbol{A}_p(r,\theta,\phi) = \frac{r_0}{r}\boldsymbol{A}_{p0}(r_0,\theta,\phi) + \frac{1}{r}\int_{r_0}^{r}\boldsymbol{B}_p(r',\theta,\phi)\times\hat{\boldsymbol{r}}\,r'\,\mathrm{d}r'$$

  with $\boldsymbol{A}_{p0}$ found using fast-Poisson solver.

- Integrate $\boldsymbol{A}_p$ along field lines with second-order Runge-Kutta method.

Toy model - single Bipolar Magnetic Region

Background | **BMR** | **Background + BMR**



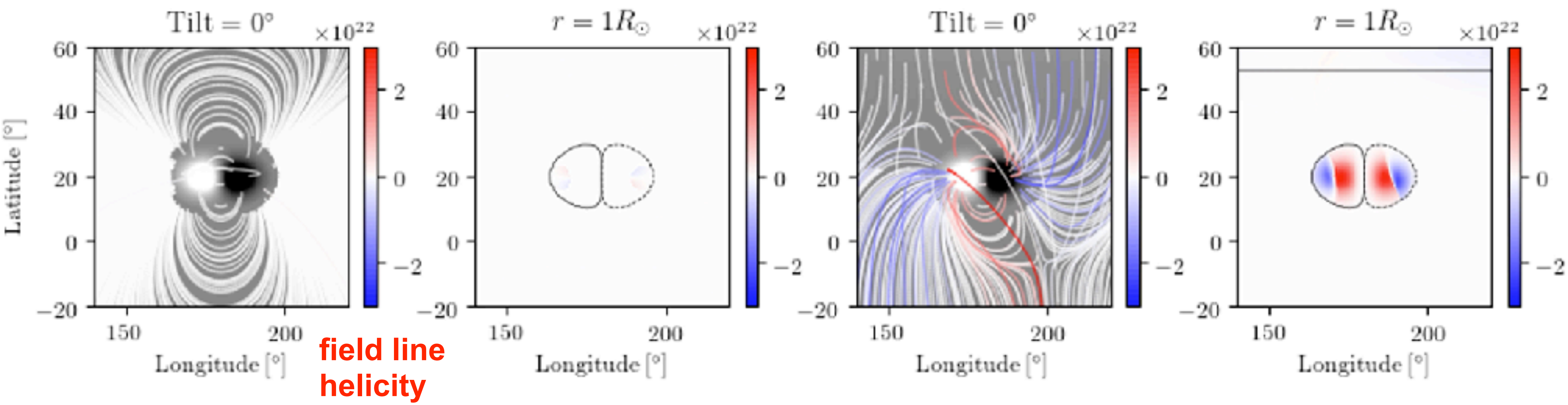
$(\mathcal{A} \equiv 0)$

$\mathcal{A} \neq 0$
only on open field lines

$\mathcal{A} \neq 0$
on open and closed field lines

Toy model

Without background | **With background**



- Helicity content is maximized when BMR is east-west.
- Helicity primarily comes from “linking” with the background field.
- There is a net helicity within an east-west BMR:

1.7×10^{42} Mx^2
-1.4×10^{42} Mx^2 → net helicity: 0.3×10^{42} Mx^2

In total:

$$\overline{H} = 4.7 \times 10^{42}\ \mathrm{Mx}^2$$

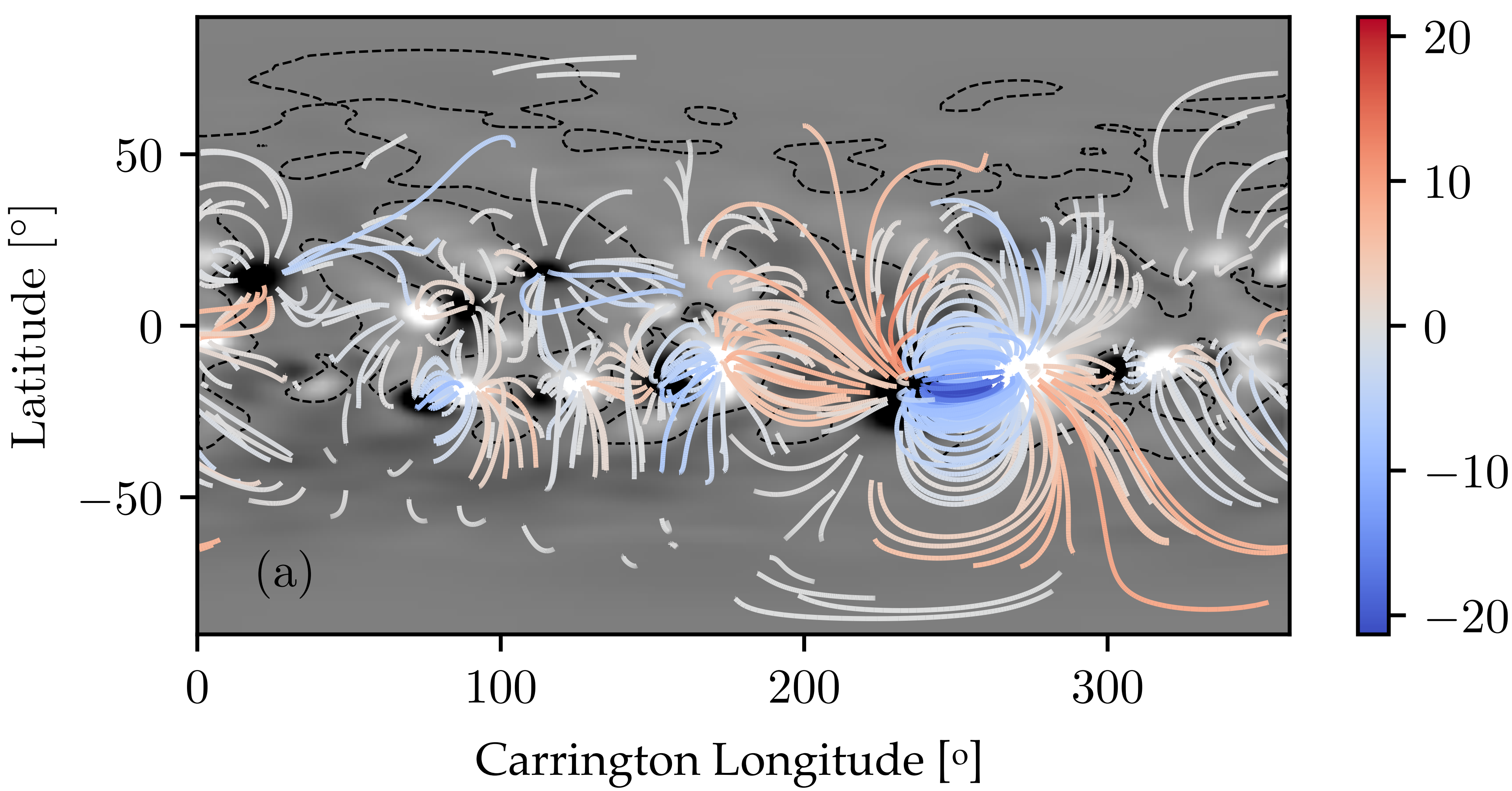
$$H = -0.07 \times 10^{42}\ \mathrm{Mx}^2$$

HMI synoptic maps

- Magnetogram data from Solar Dynamics Observatory/Helioseismic and Magnetic Imager.
- Radial component pole-filled synoptic maps [Sun 2018].
- Carrington Rotation 2098 (June 2010) to 2226 (February 2020).
- Spherical harmonic smoothing filter.

e.g. CR 2157

$\mathcal{A}|B_r|$

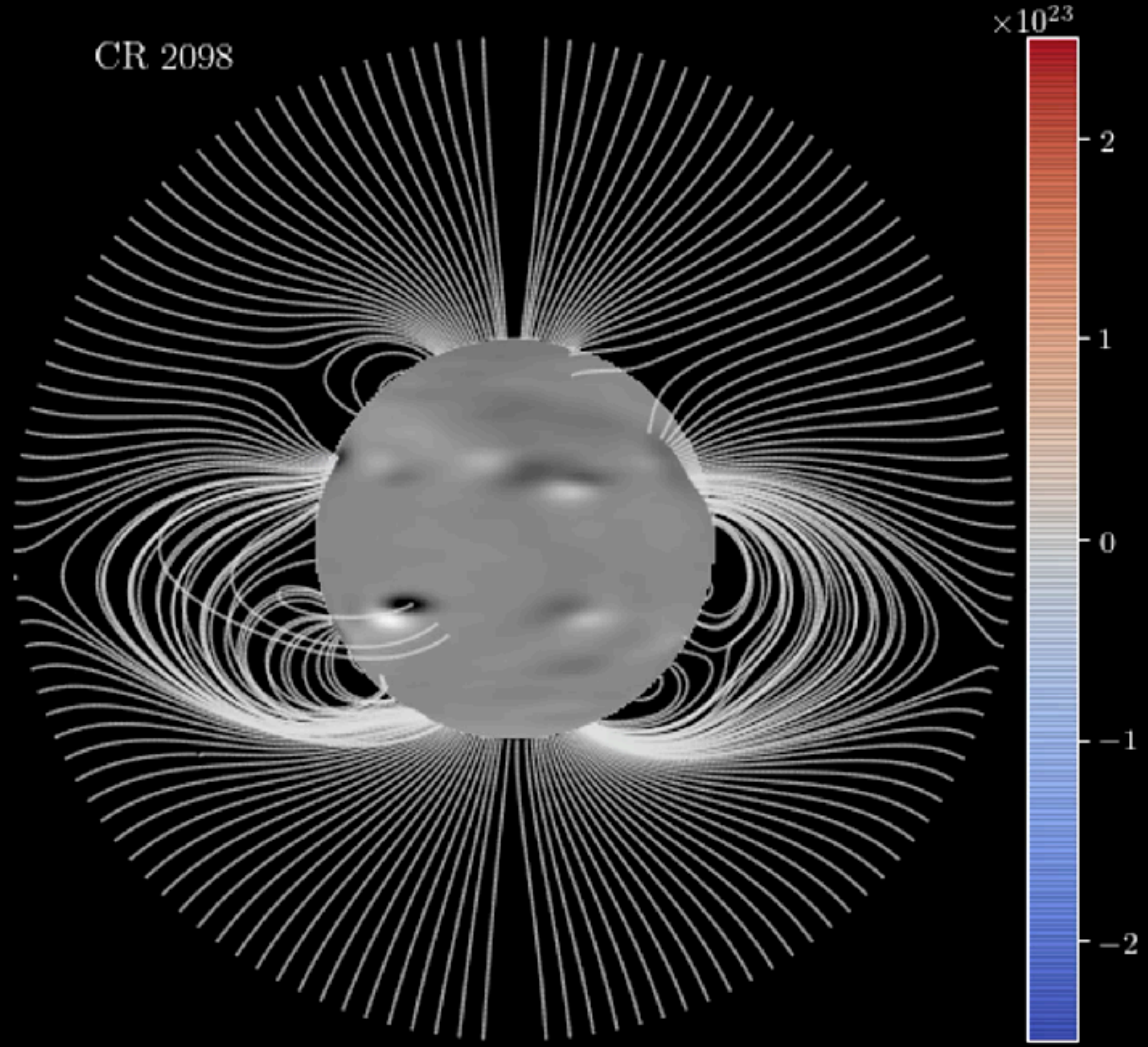


$b_0 = 2 \times 10^{-5}$

$b_0 = 1 \times 10^{-4}$

$b_0 = 5 \times 10^{-4}$

HMI synoptic maps



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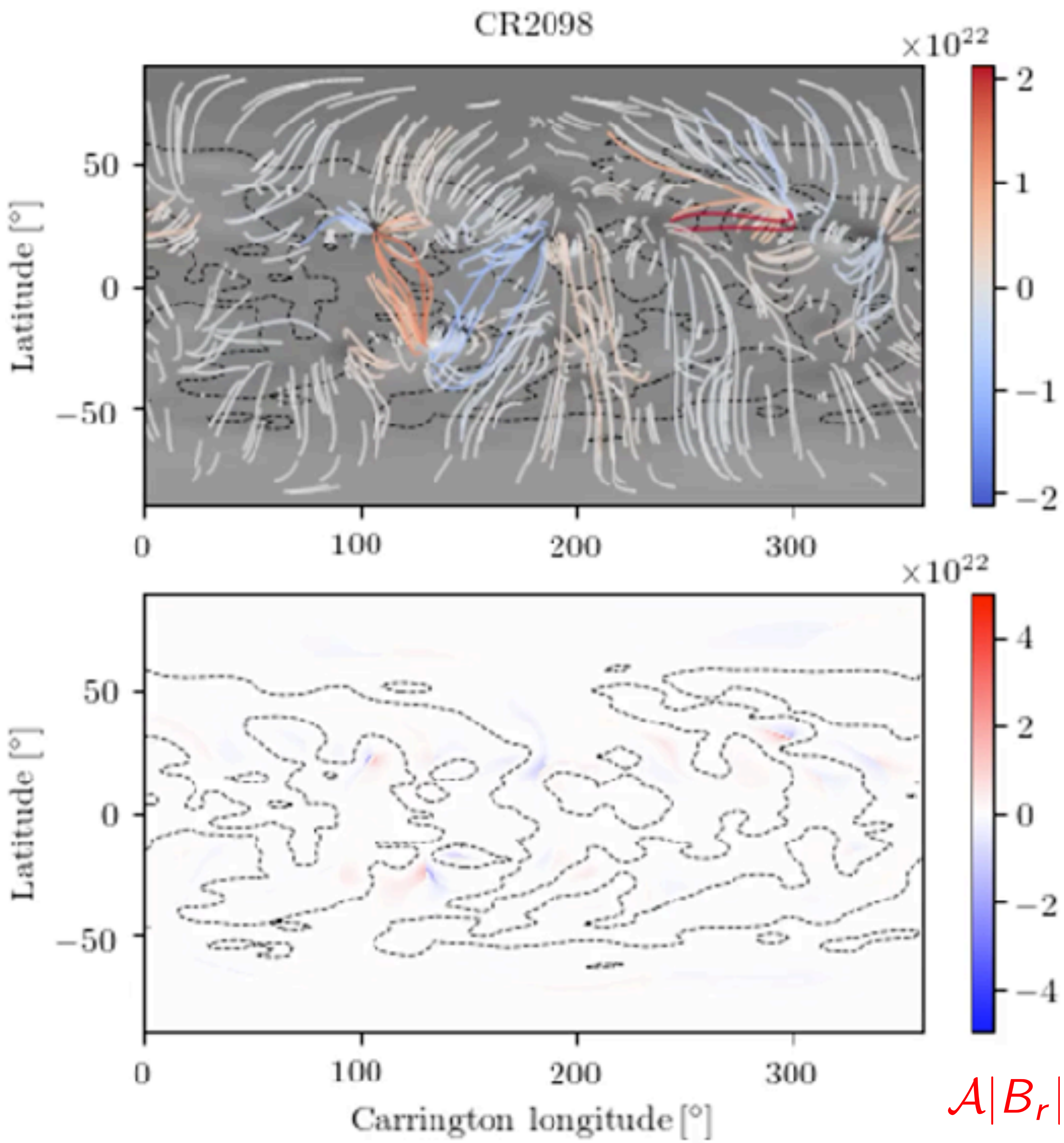
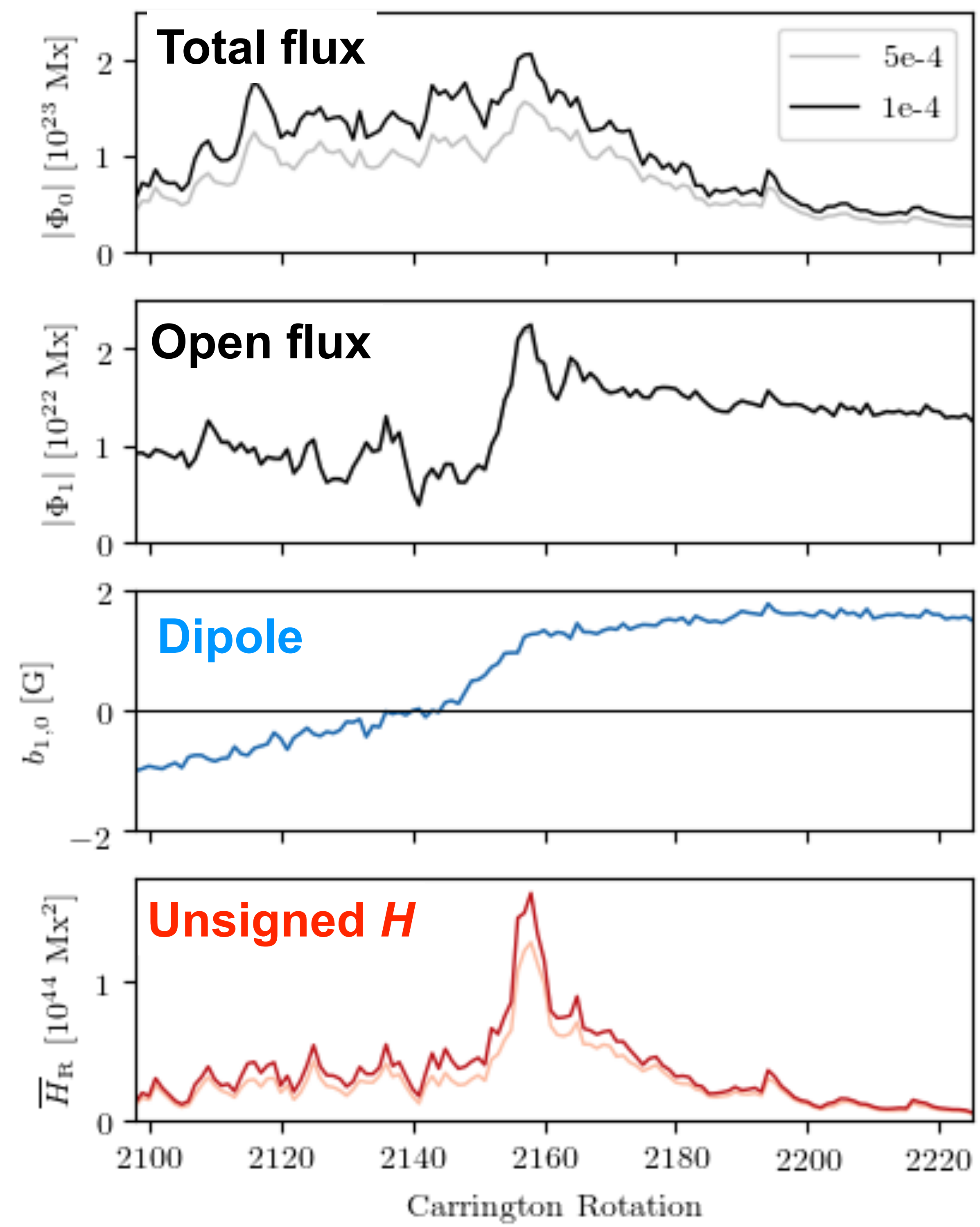
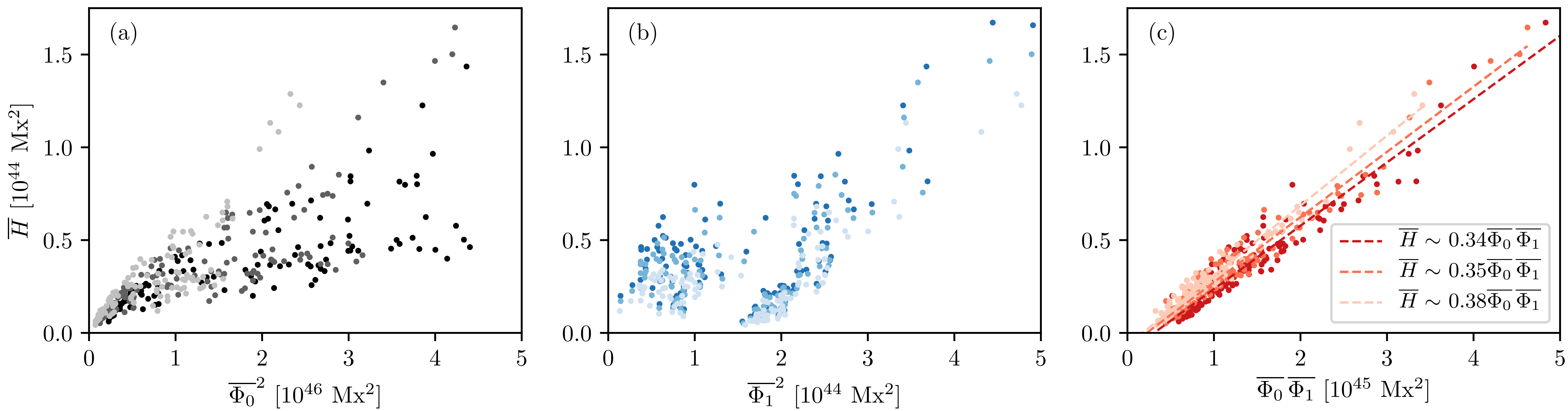




- Helicity is predominantly in the active region belts.
- Total helicity doesn't correlate directly with total flux…

HMI synoptic maps

Unsigned *H*

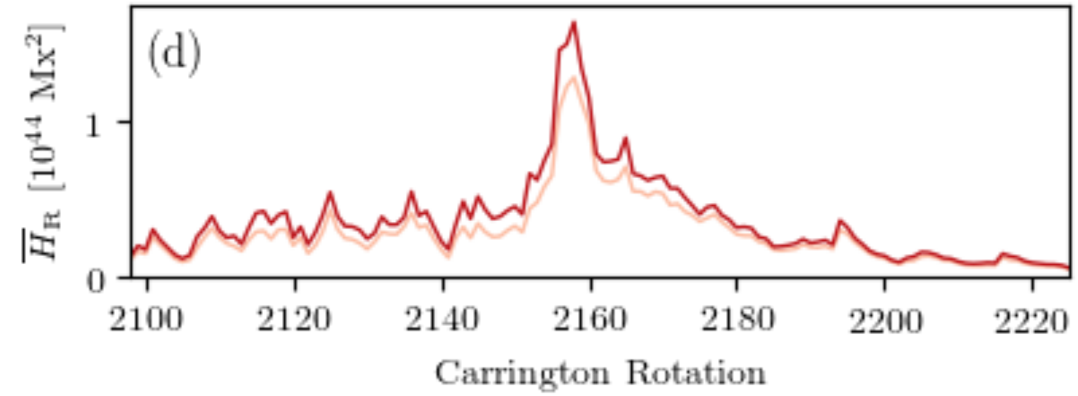


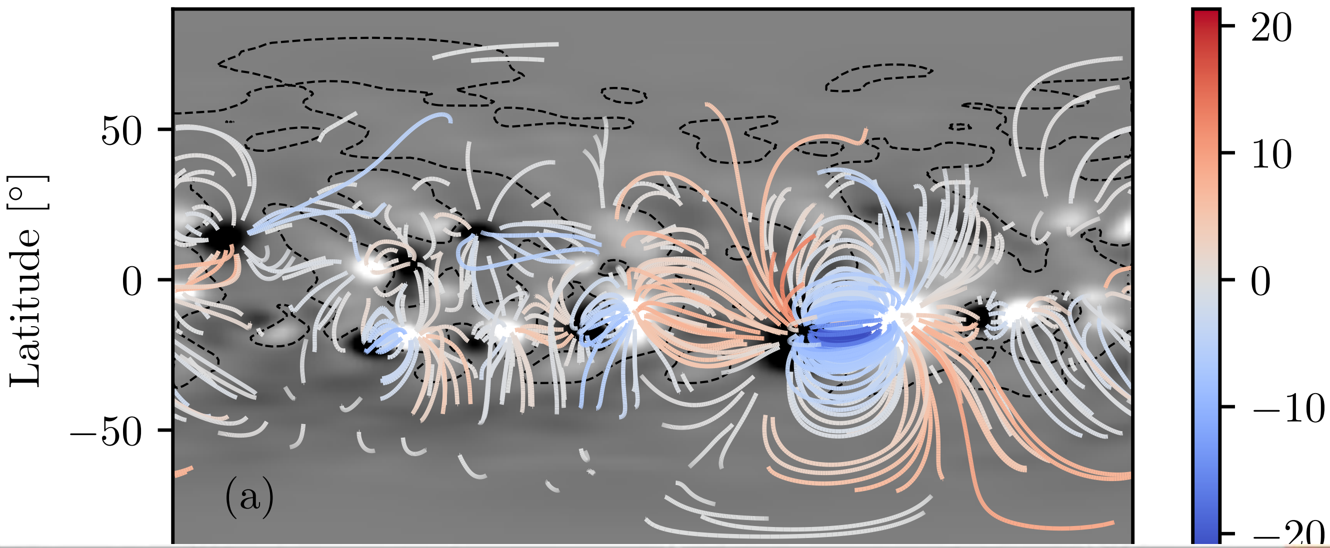
(Total flux)² **(Open flux)²** **(Total flux)*(Open flux)**

- Suggests that helicity mostly arises from linking of active region flux with overlying field.

Cause of the peak?

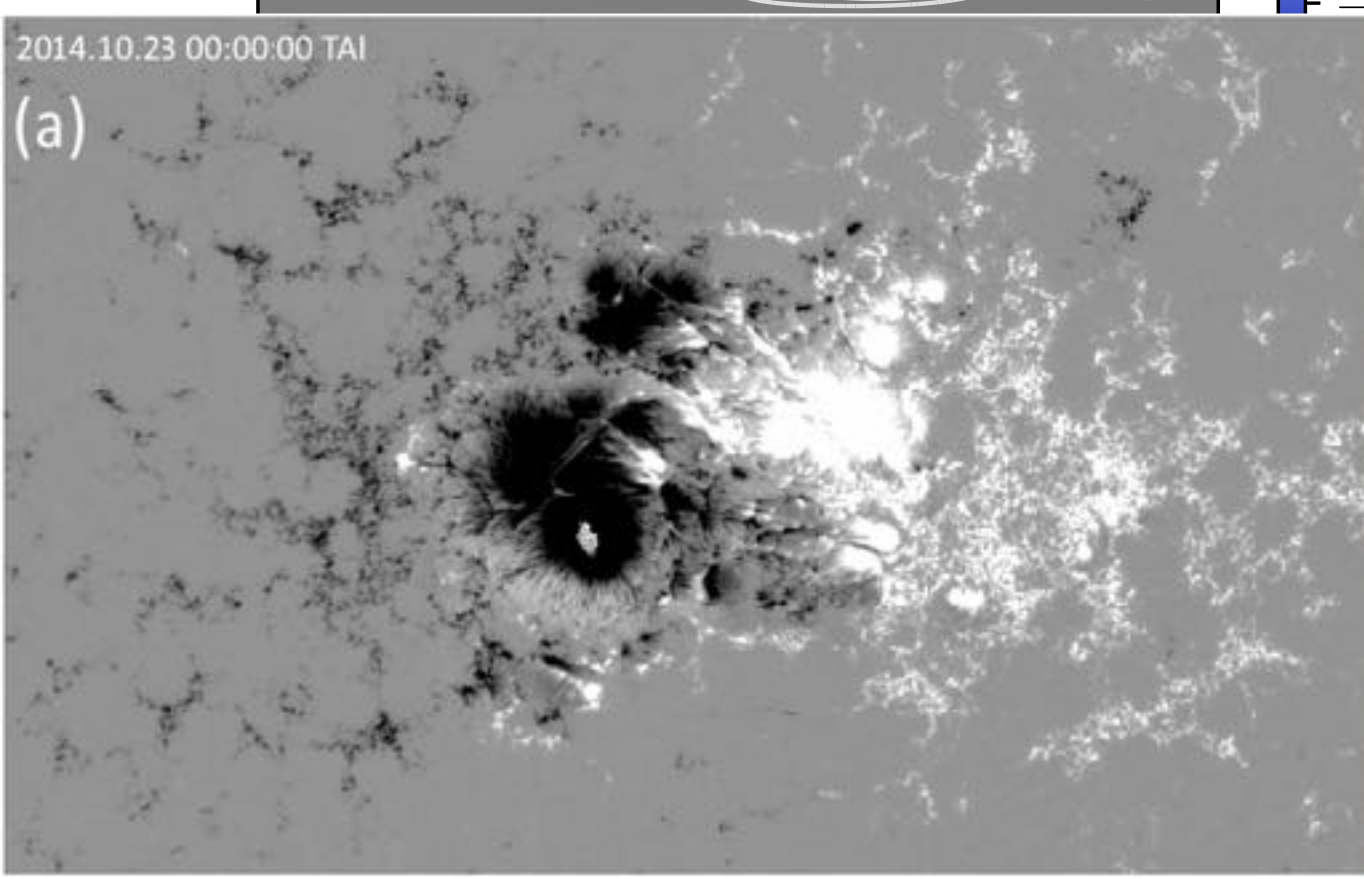
- The cause of the large peak is a single strong active region NOAA 12192. [Sun et al. 2015]

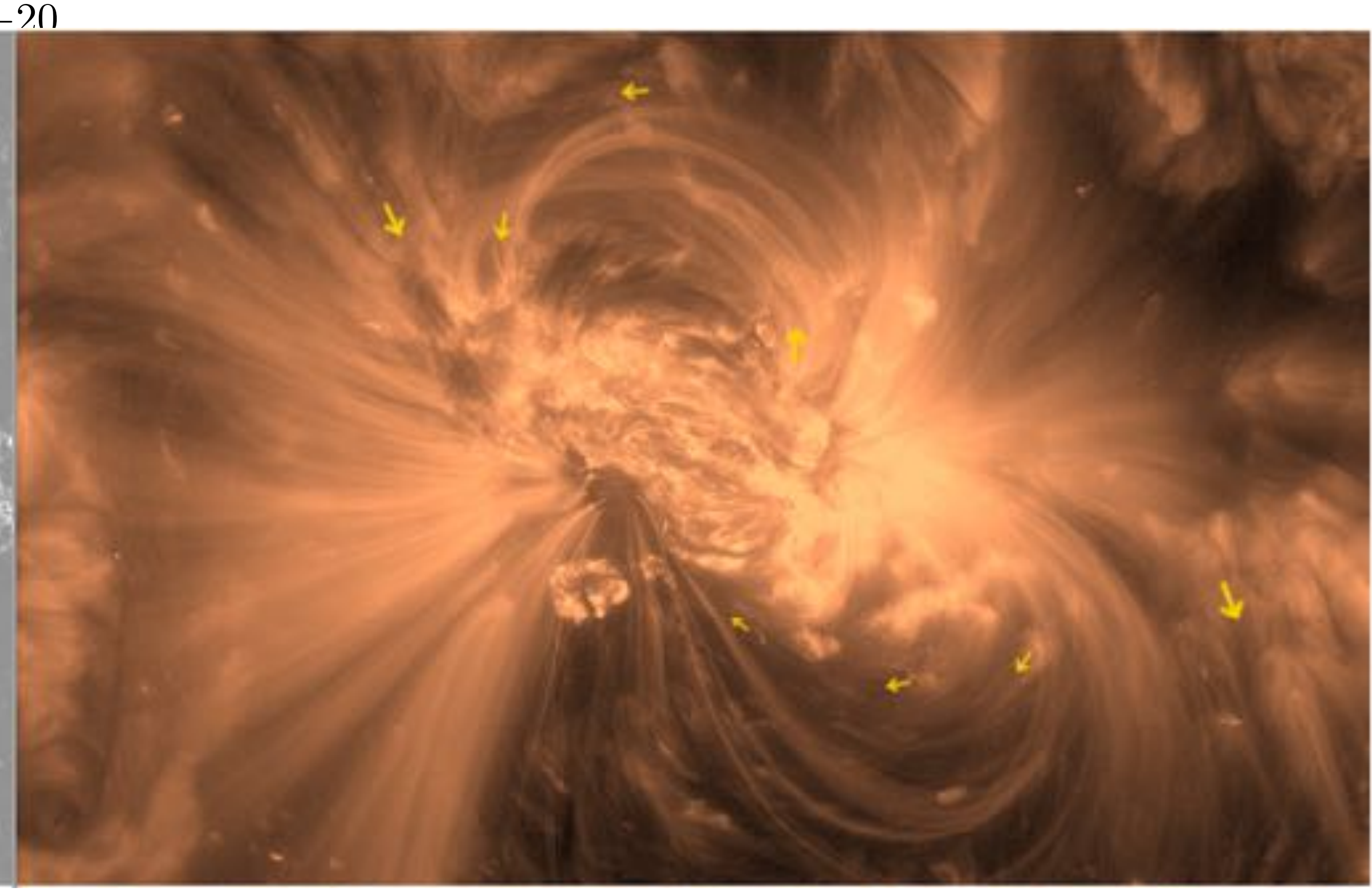




- Has negative *H* because it emerges **after** polar field reversal with **positive** leading polarity.

[cf. McMaken-Petrie *ApJ*, 2017] - chirality of EUV loops





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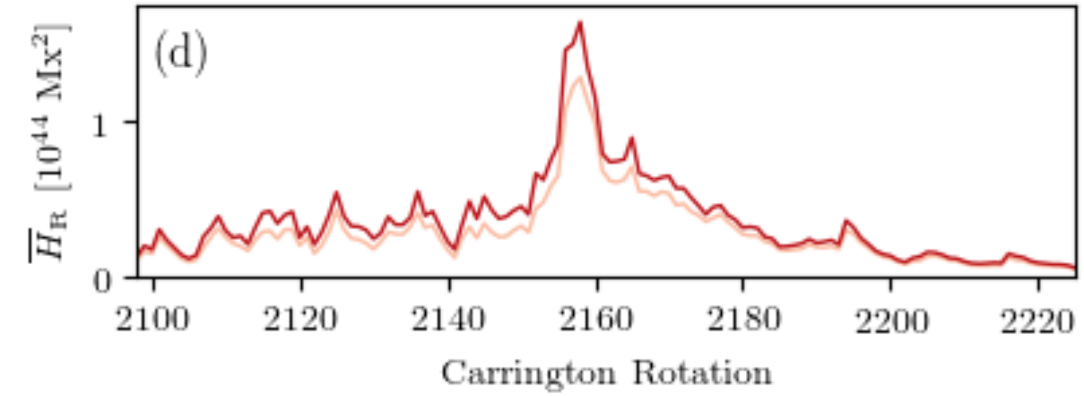
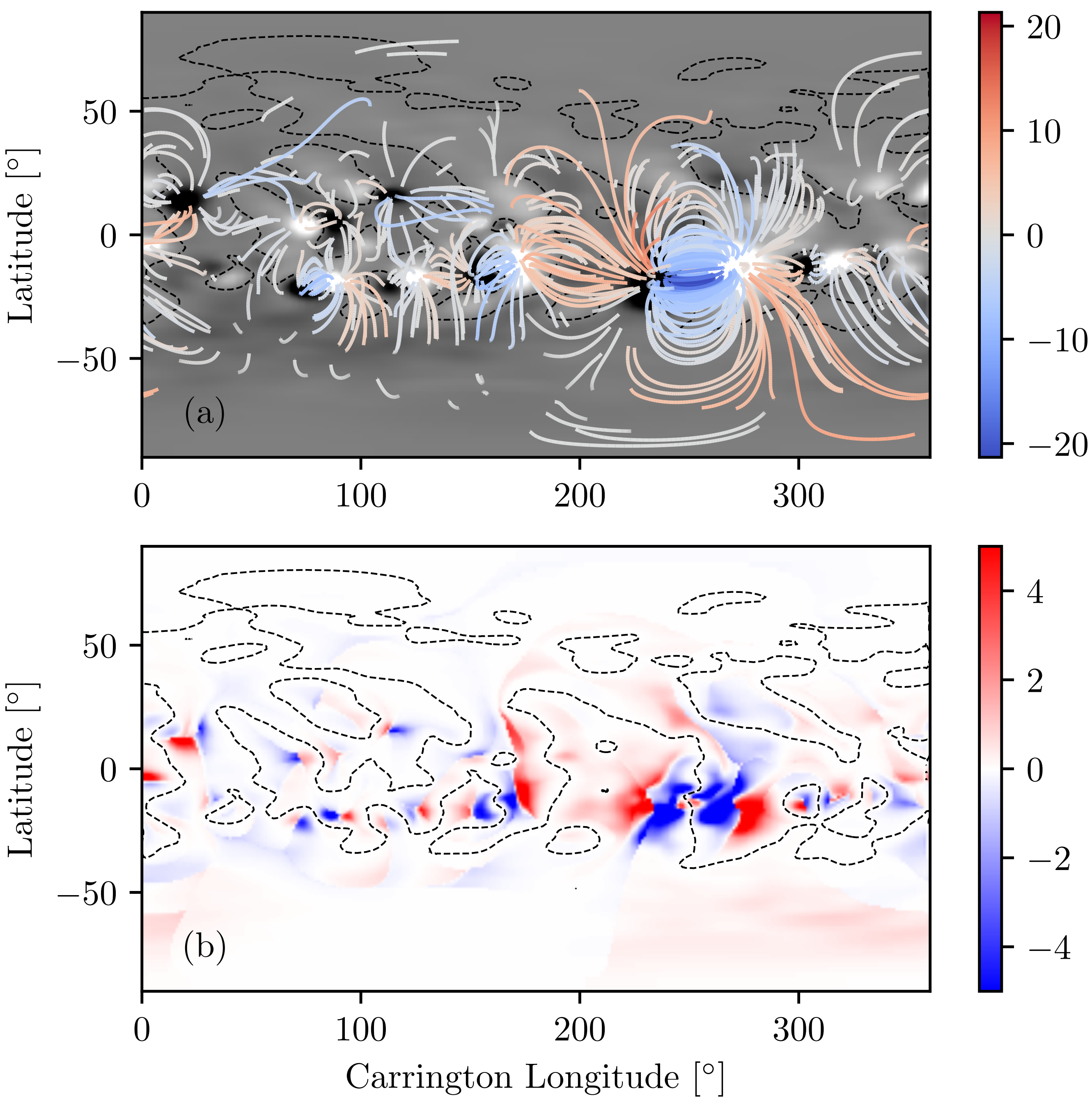
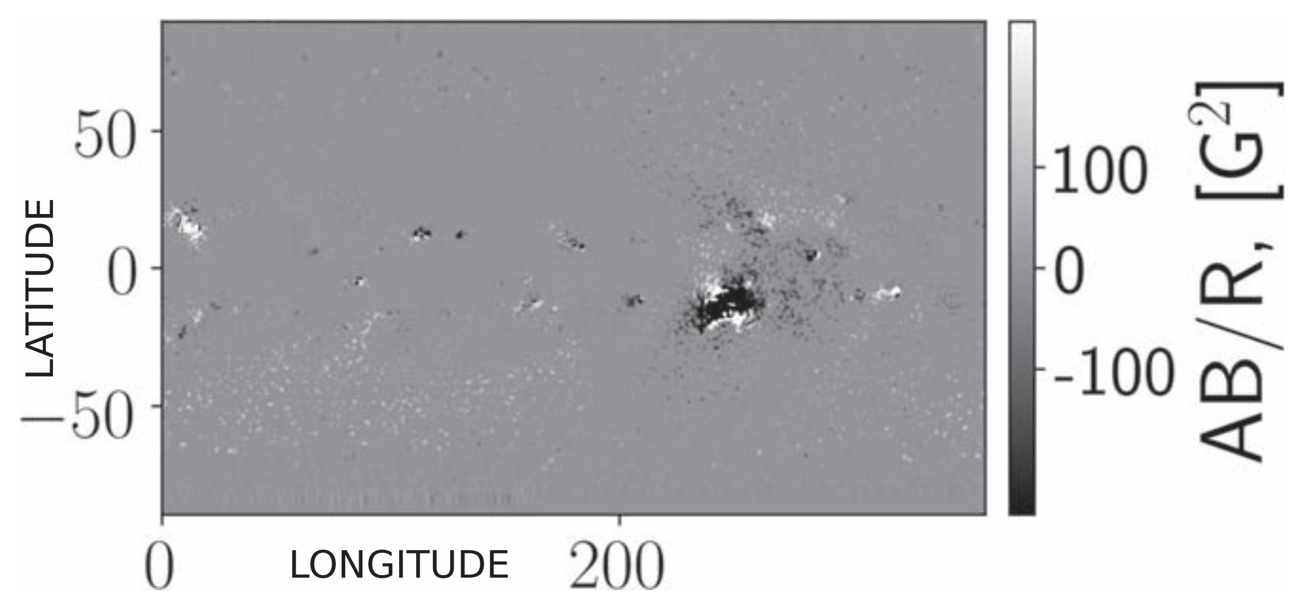




- Has negative *H* because it emerges **after** polar field reversal with **positive** leading polarity.

[cf. McMaken-Petrie *ApJ*, 2017] - chirality of EUV loops

[cf. Pipin et al *ApJL*, 2019] - estimate ***A.B*** from vector magtms:

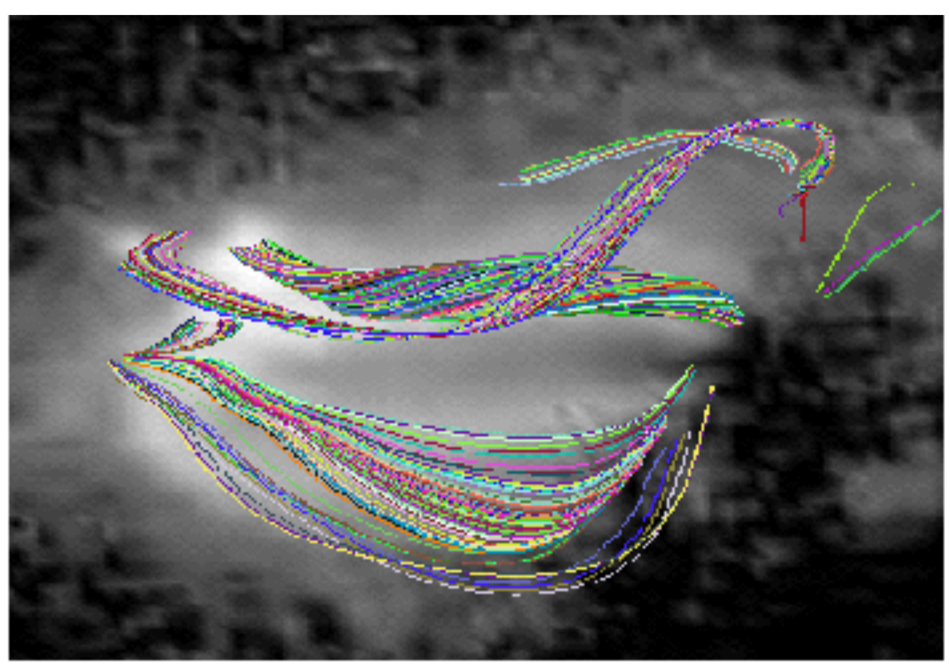


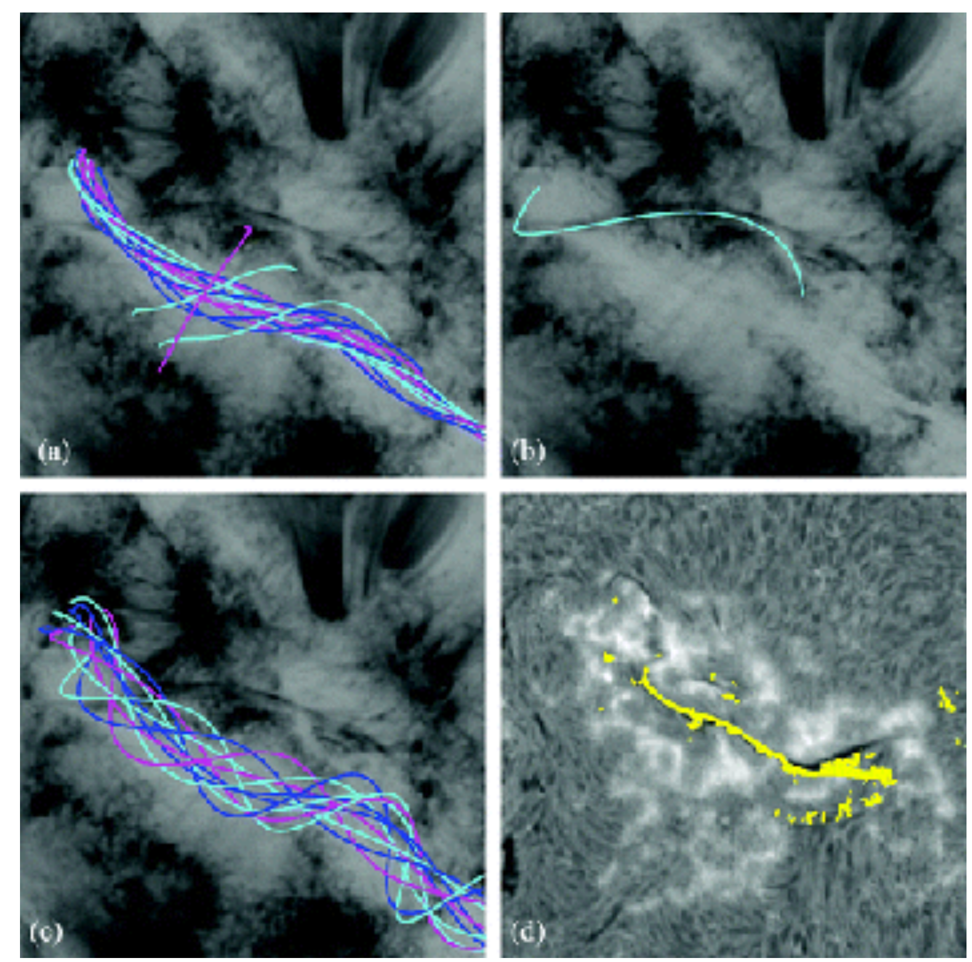
In context

- Time-average over global **potential field** (high-res): $\overline{H} = 4 \times 10^{43}\,\mathrm{Mx}^2$

- Typical (relative) helicity of a significant **non-potential** active region:

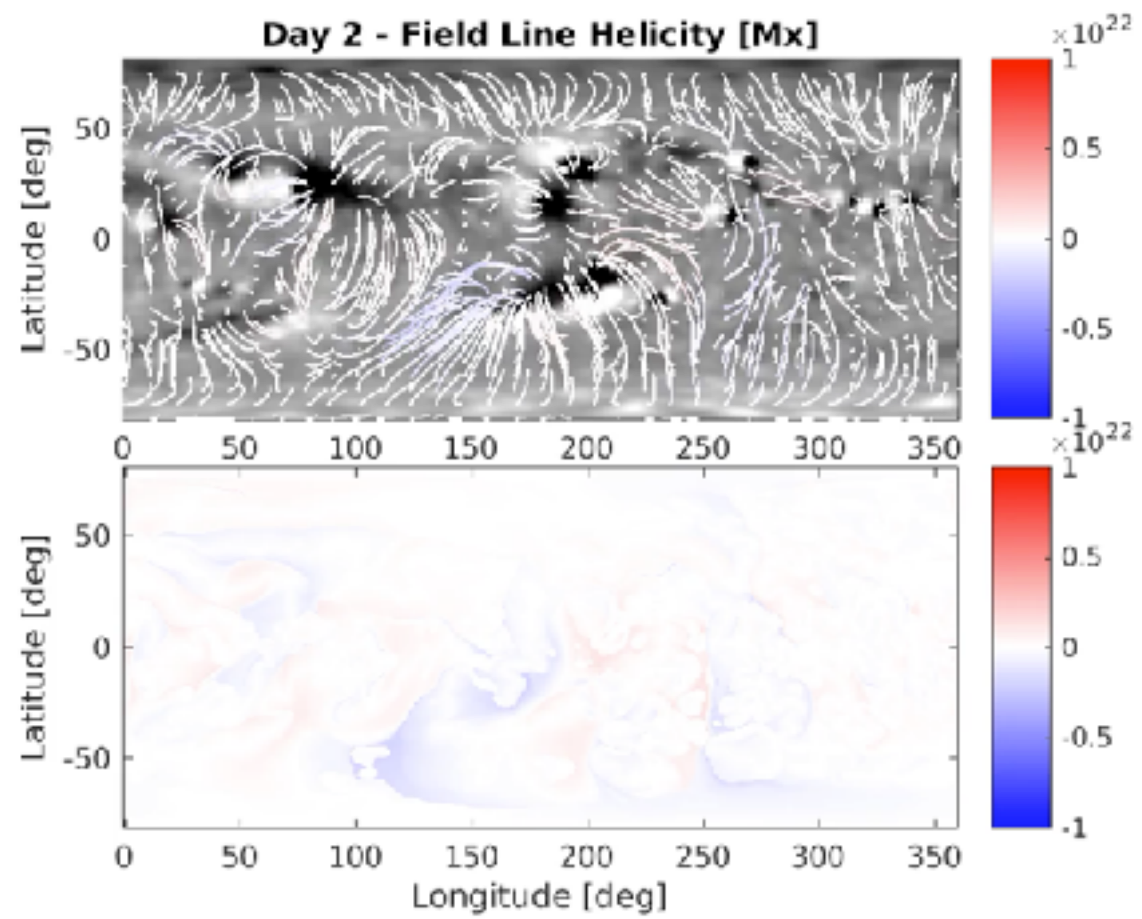
  $\approx 2 \times 10^{43}\,\mathrm{Mx}^2$ [DeVore *ApJ*, 2000, Bleybel et al. *A&A* 2002, Bobra et al. *ApJ* 2008, Pevtsov *JApA* 2008, Georgoulis et al *ApJL* 2009]





Conclusion

- Potential fields in the solar corona contain (field line) helicity.
- It predominantly arises from linking of active regions with overlying magnetic field.
- The total *absolute* helicity content is comparable to 2 non-potential active regions.
- The net helicity content is zero globally but can be unbalanced within an active region.
- Can be imprinted on non-potential field, e.g. acting as seed for amplification by photospheric shearing flows [Yeates-Hornig *A&A* 2016]



- More details:

  Yeates, The Minimal Helicity of Solar Coronal Magnetic Fields, *ApJL* **898** L49 (2020)

  - and references therein

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