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Revisiting Taylor relaxation

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Aims

- Resistive relaxation of plasma with initially "braided" magnetic field shows spontaneous self-organization.

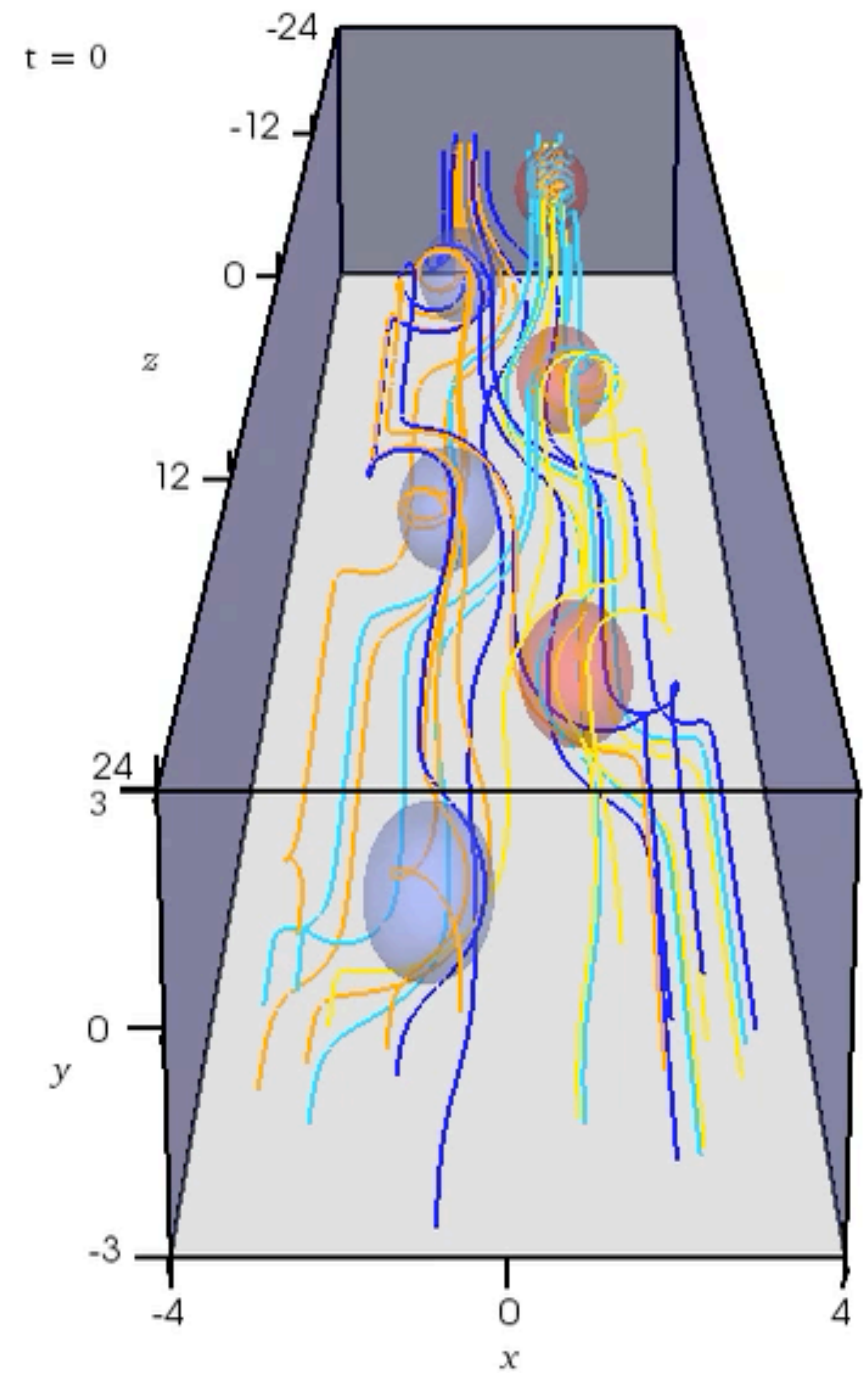
- Relaxed state not predicted by **Taylor relaxation** theory: conservation of total magnetic helicity => linear force-free field $\nabla \times \mathbf{B} = \lambda_0 \mathbf{B}$.

  [Taylor, *Rev Mod Phys* **58**, 741, 1986]

- Can we learn more by following the evolution of **field line helicities**?

$$\mathcal{A}(L) = \lim_{\epsilon \to 0} \frac{\int_{V_\epsilon(L)} \mathbf{A} \cdot \mathbf{B} \, dV}{\Phi(V_\epsilon(L))} = \int_L \mathbf{A} \cdot d\mathbf{l}$$

  [Berger, *Astron Astrophys* **201**, 355, 1988;
  Yeates & Hornig, *Phys Plasmas* **20**, 012102, 2013;
  Aly, *Fluid Dyn Res* **50**, 011408, 2018]



- Taylor assumed that the **A.B** would be arbitrarily redistributed between field lines so individual field line helicities play no role in determining the final state.

Simulation setup

- Resistive-MHD equations in Cartesian domain [-8,8] x [-8,8] x [-24, 24].

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{[viscosity]}$$

$$\rho \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \mathbf{j})$$

$$\rho \frac{\mathrm{D}\epsilon}{\mathrm{D}t} = -p \nabla \cdot \mathbf{v} + \eta |\mathbf{j}|^2 + \text{[viscous disspn.]}$$

$$p = \rho \epsilon (\gamma - 1) \qquad \beta \approx 0.01 \qquad \gamma = \tfrac{5}{3}$$

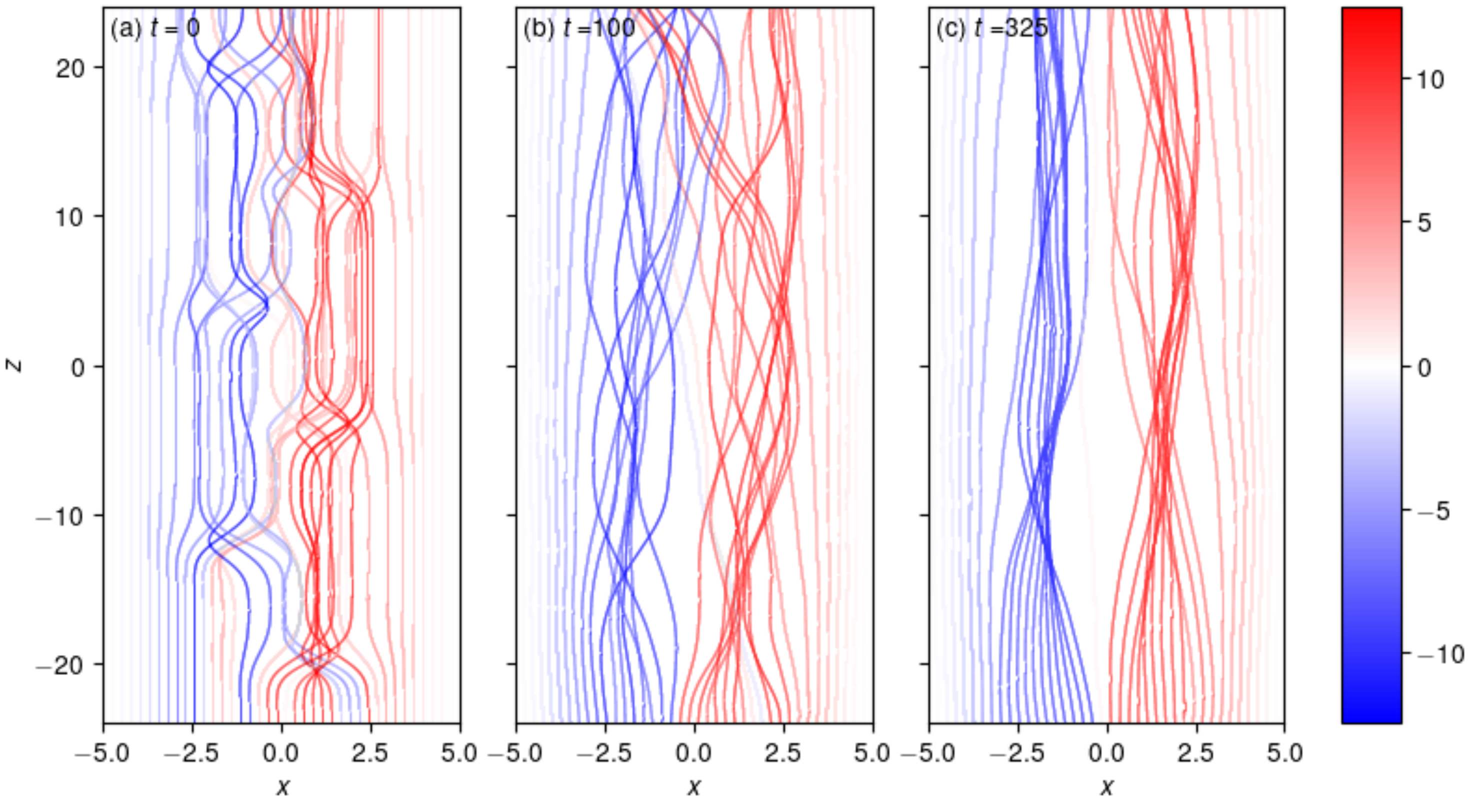
$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$

- Lundquist number $S = \eta^{-1}$ from 2,500 to 20,000.
- Line-tied boundaries $\mathbf{v} = \mathbf{0}$
- Initially braided magnetic field.
- LARE3d code (T. Arber). https://github.com/Warwick-Plasma/Lare3d

Evolution of field line helicity

- Compute **A** and integrate along field lines for a sequence of snapshots.



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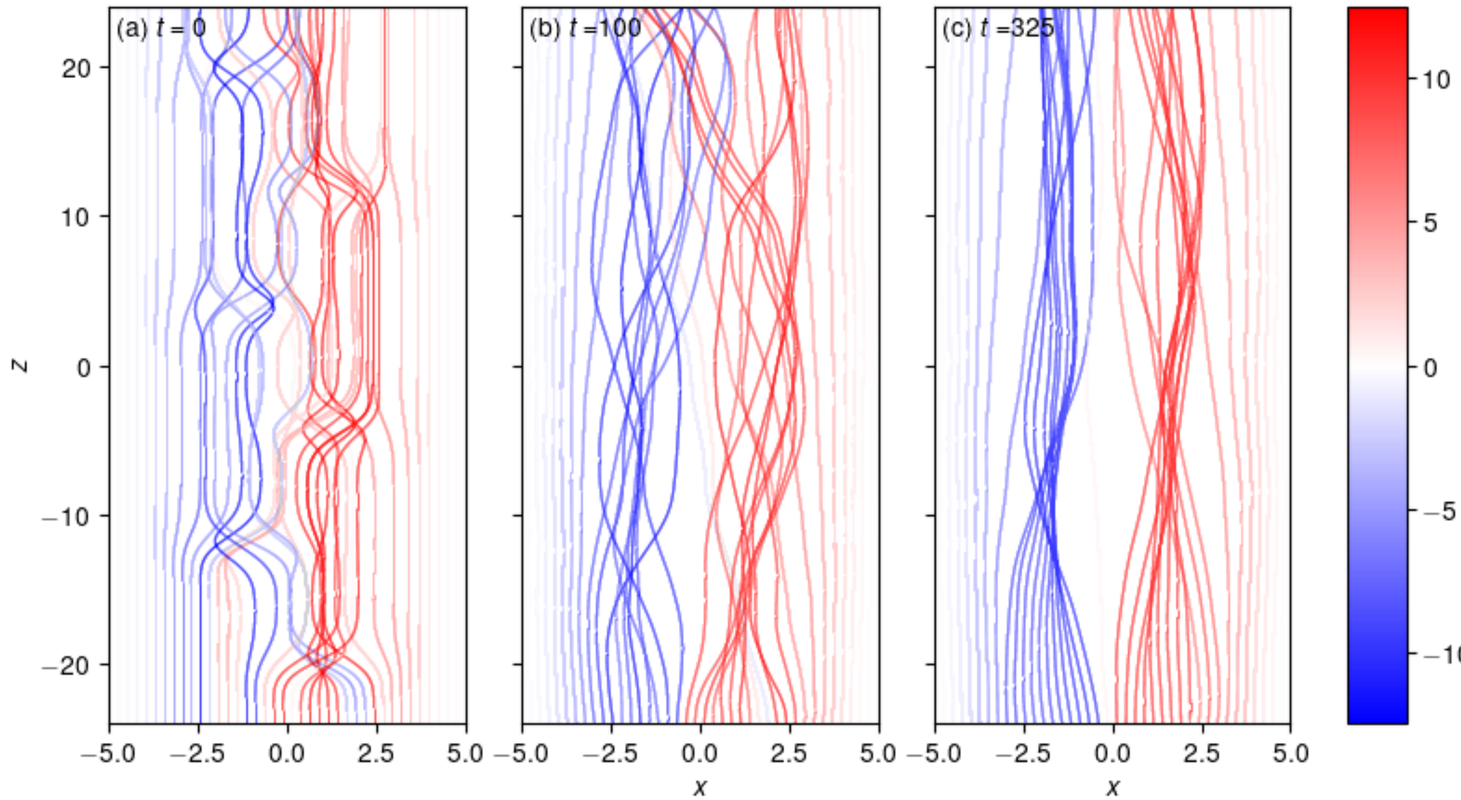
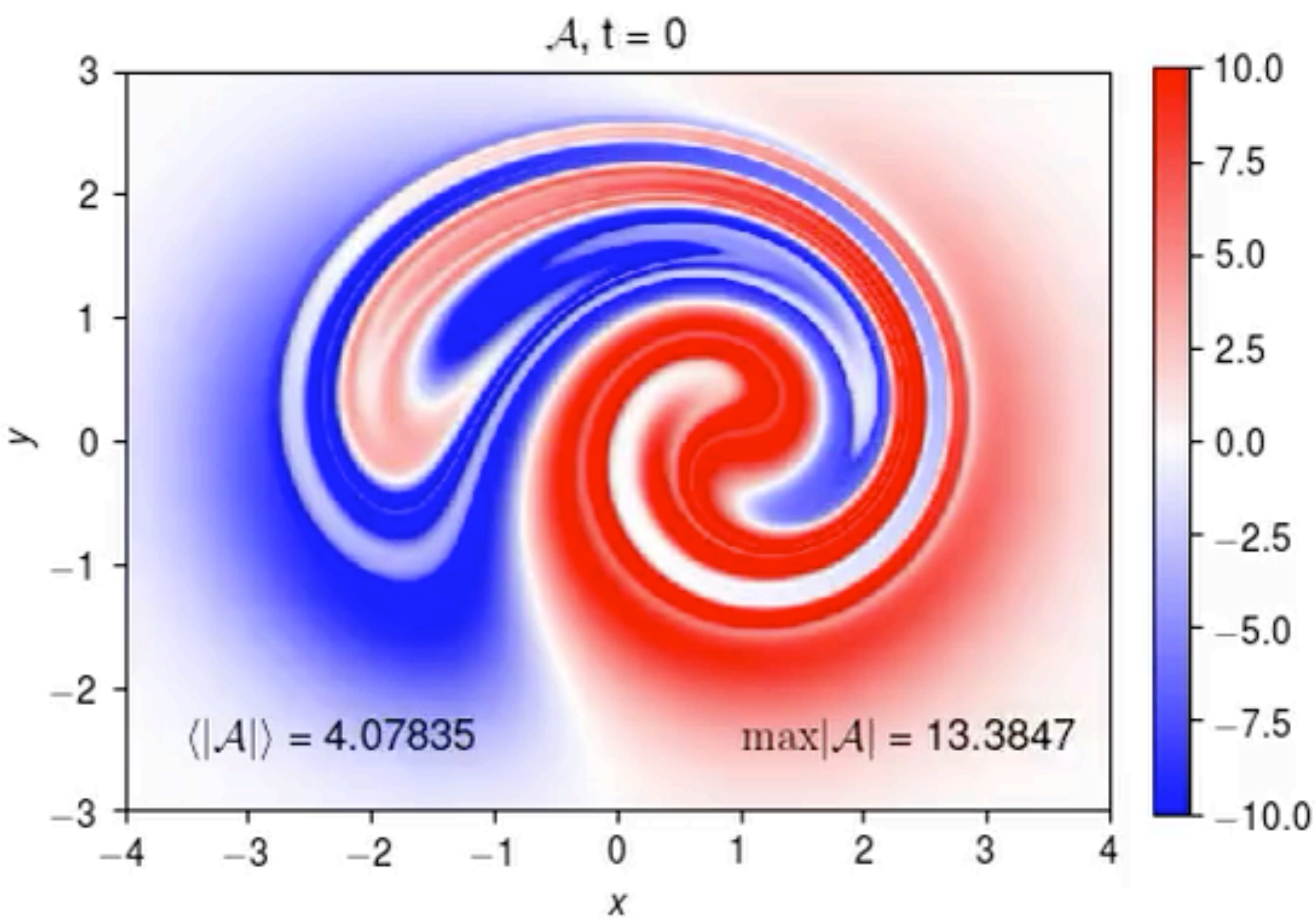


- Cross-section on the lower boundary:



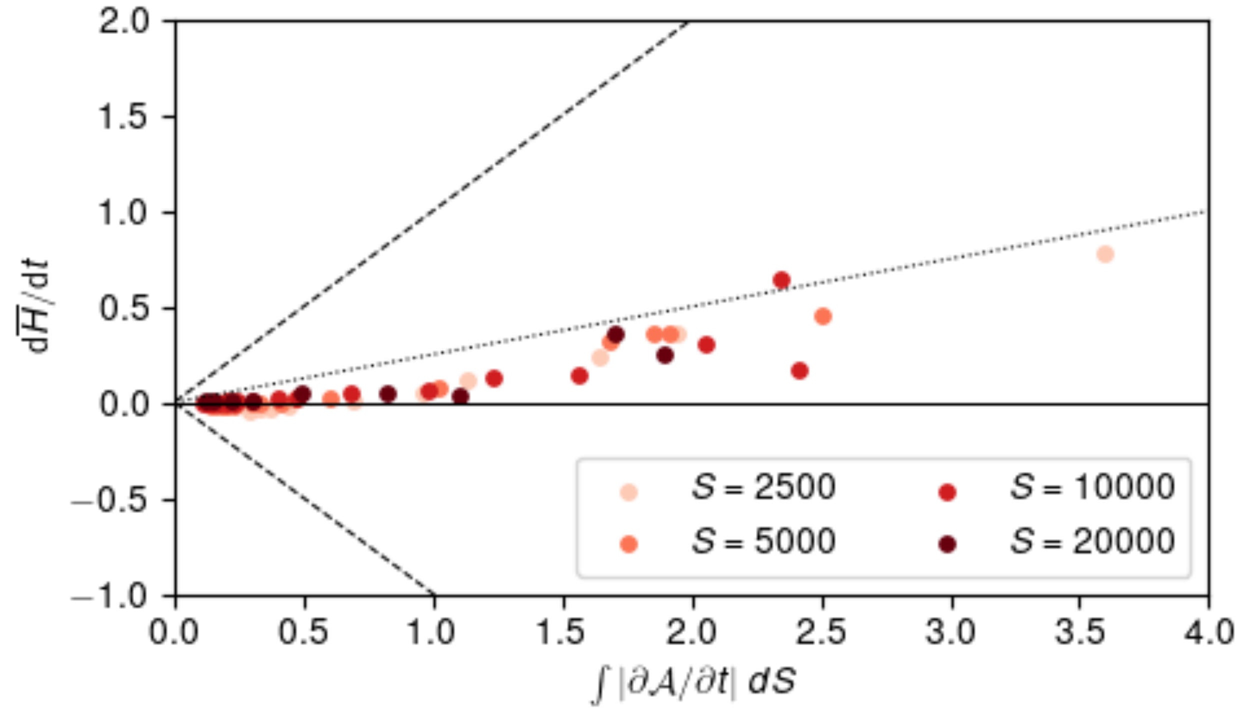
1. To leading order, field line helicity is **redistributed** rather than destroyed.
2. The relaxed state exhibits **self organization** into distinct positive and negative regions.
3. Within each of these regions, the field line helicity is strikingly **uniform**.

1. Dominance of redistribution

- Verifies our earlier prediction that evolution of field line helicity in high Rm is dominated by redistribution rather than dissipation.

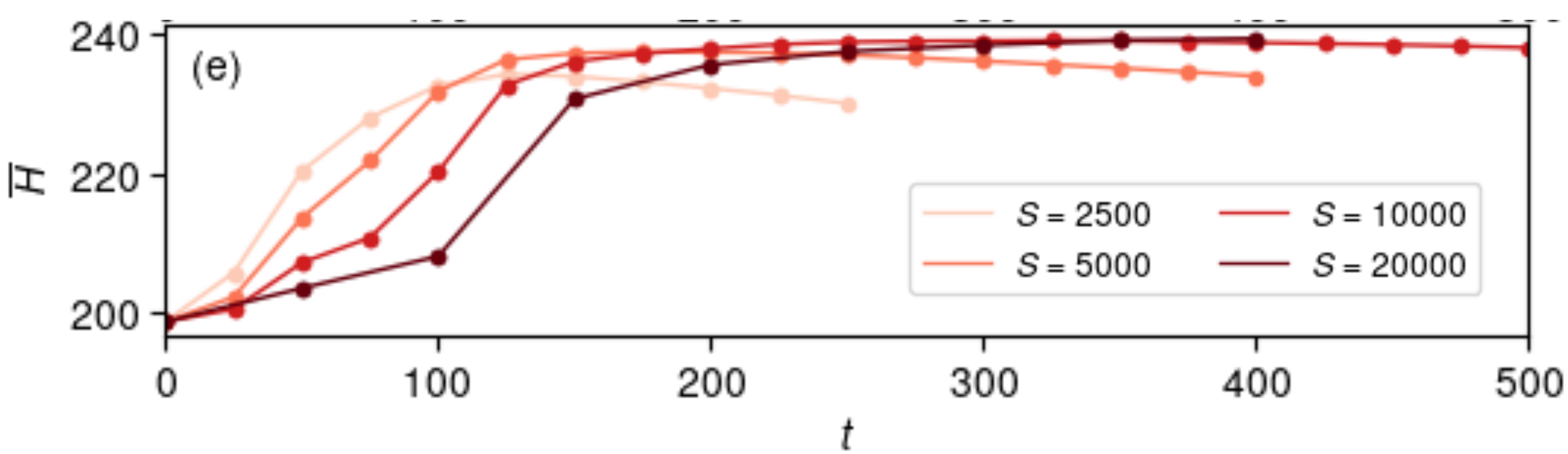
  [Russell et al., *Phys Plasmas* **22**, 032106, 2015]

$$\overline{H} = \int_{-4}^{4} \mathrm{d}x \int_{-4}^{4} \mathrm{d}y \left(|\mathcal{A}| B_z \right)_{z=-24}$$



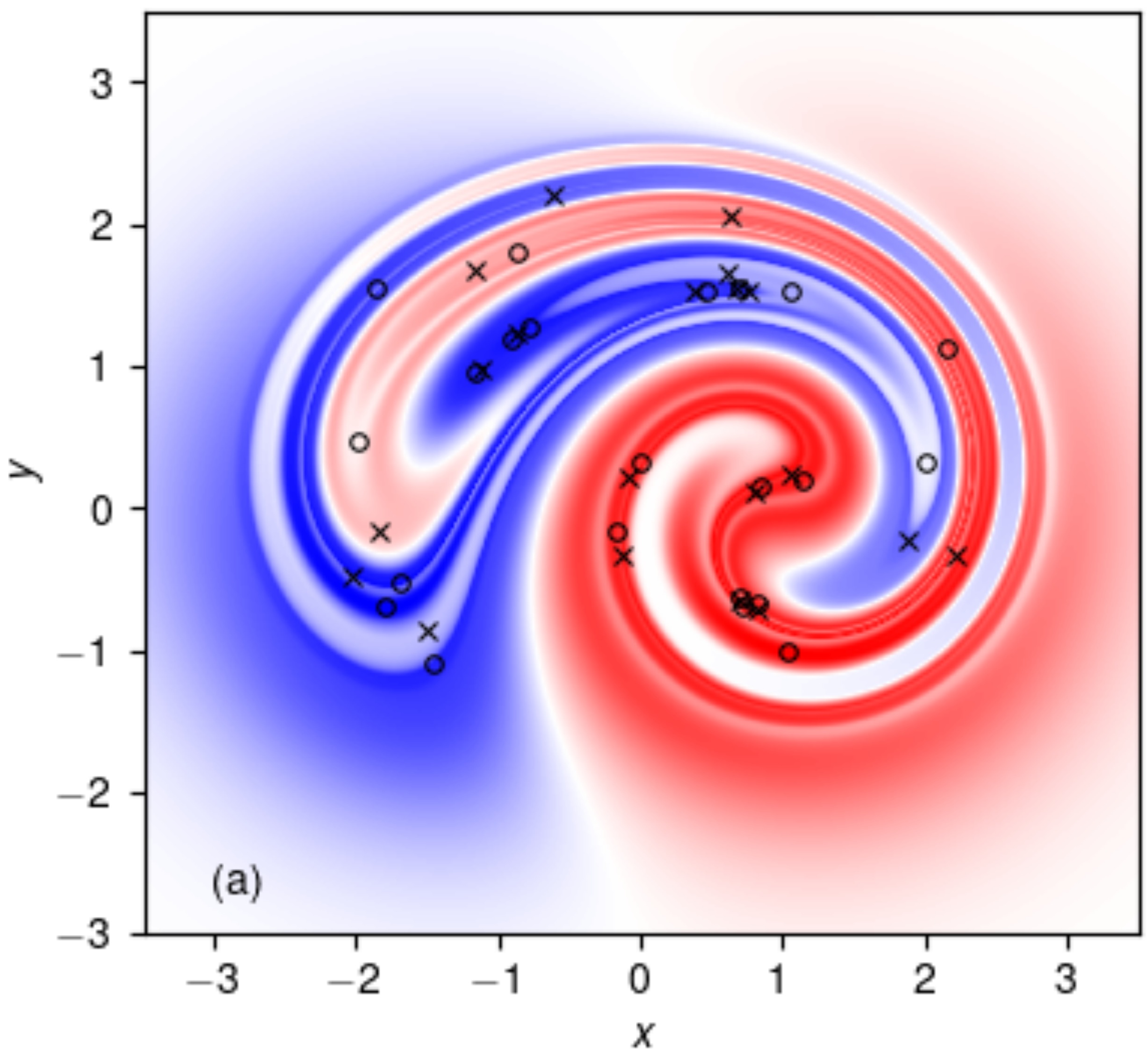
[See upcoming paper for details of the evolution equation that led to this prediction.]

- Notice that the small changes in $\overline{H}$ do have a net positive drift:



2. Self organization

- Changes in FLH are localized, so the total Poincaré index of critical points (where $\nabla\mathcal{A} = \mathbf{0}$) is invariant.



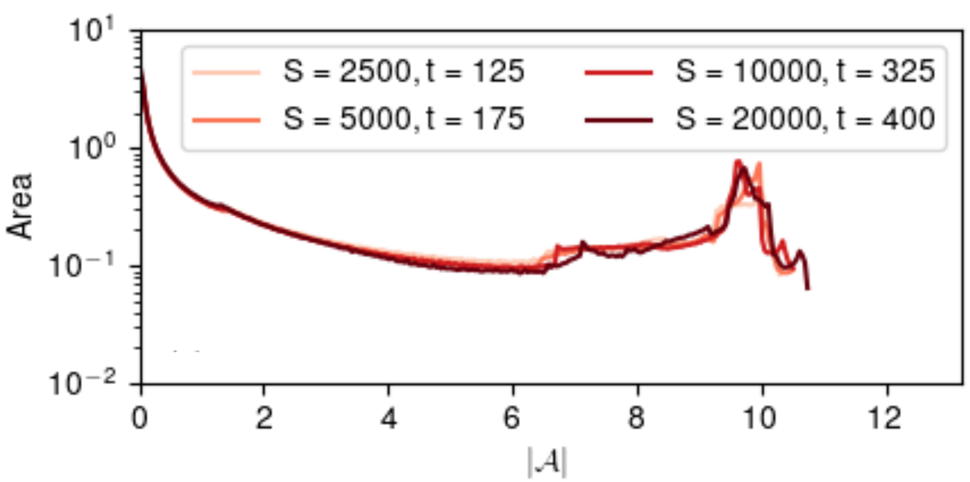
- Initially +2 (+22, -20) — explains two final regions (one maximum, one minimum).

3. Uniformization

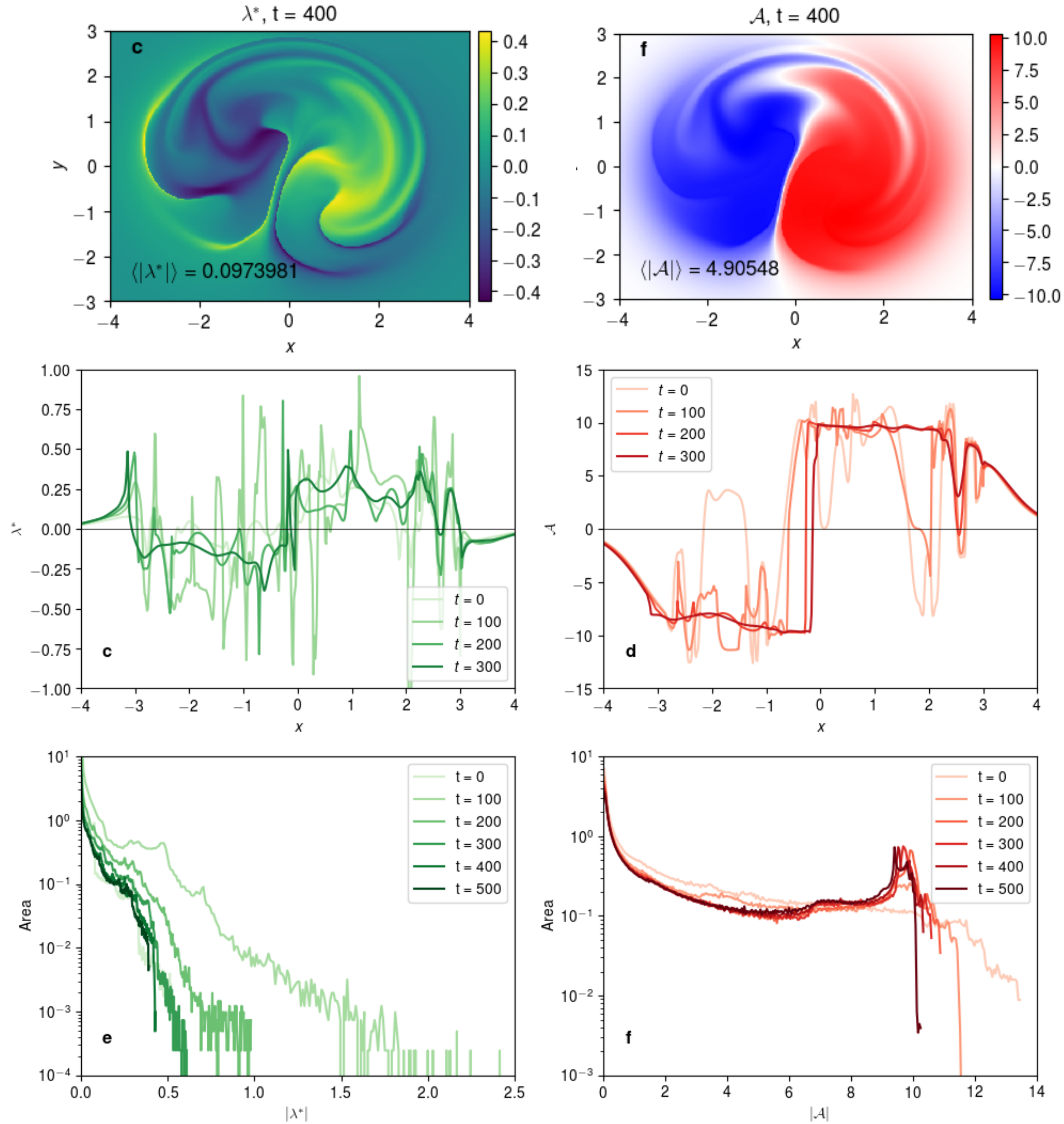
- Field line helicity is much more uniform than the “force-free parameter”,

$$\lambda = \frac{\mathbf{j}\cdot\mathbf{B}}{|\mathbf{B}|^2}$$

- The relaxed state appears independent of Lundquist number.



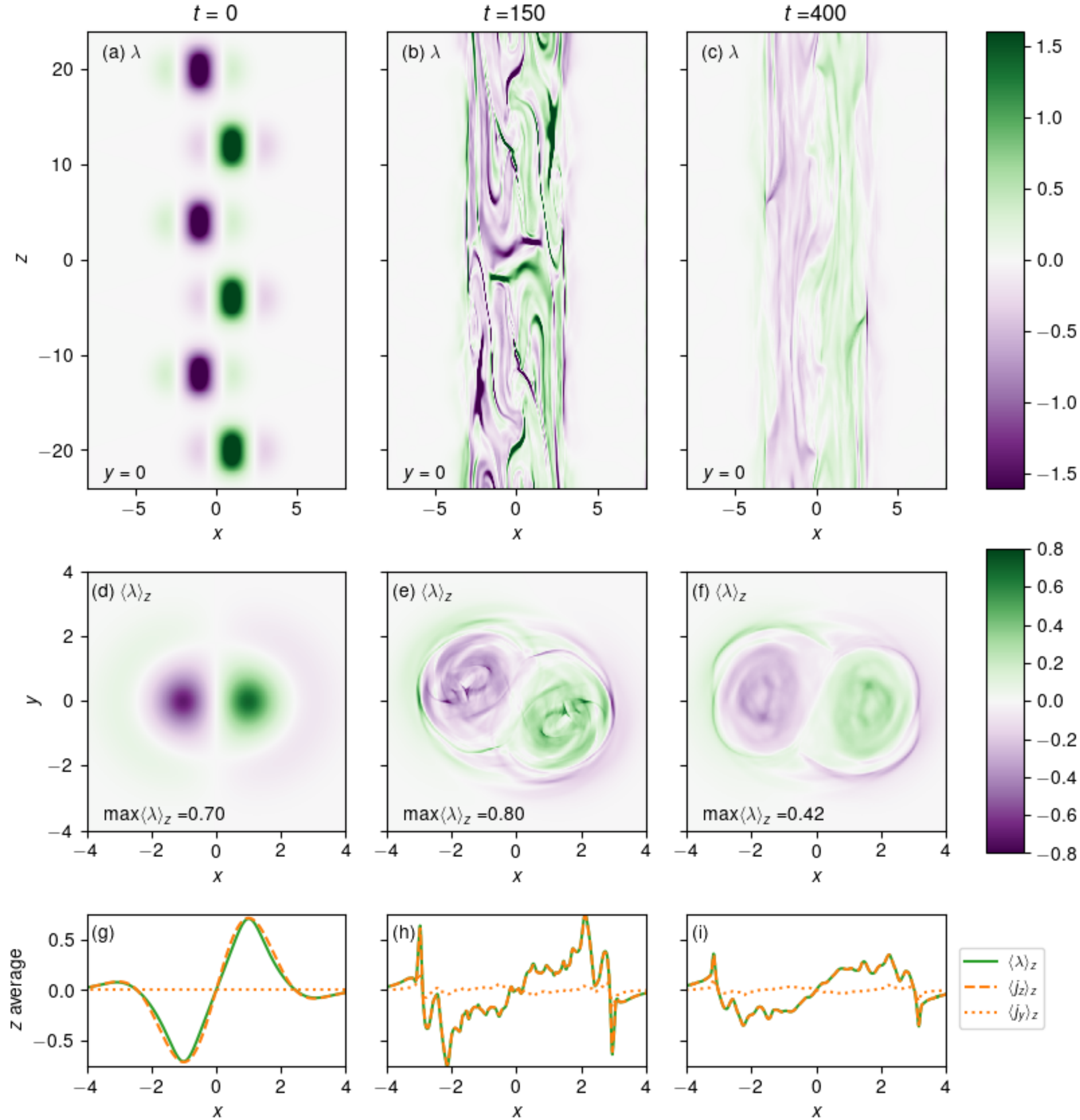
Force-free parameter | **Field line helicity**



Hypothesis

- There *is* a tendency for Taylor relaxation within each tube, with
  $\lambda \approx$ constant
  to (rough) first approximation.
- With our strong guide field, $\lambda \approx j_z$.
- An axisymmetric field with constant j_z has uniform field line helicity

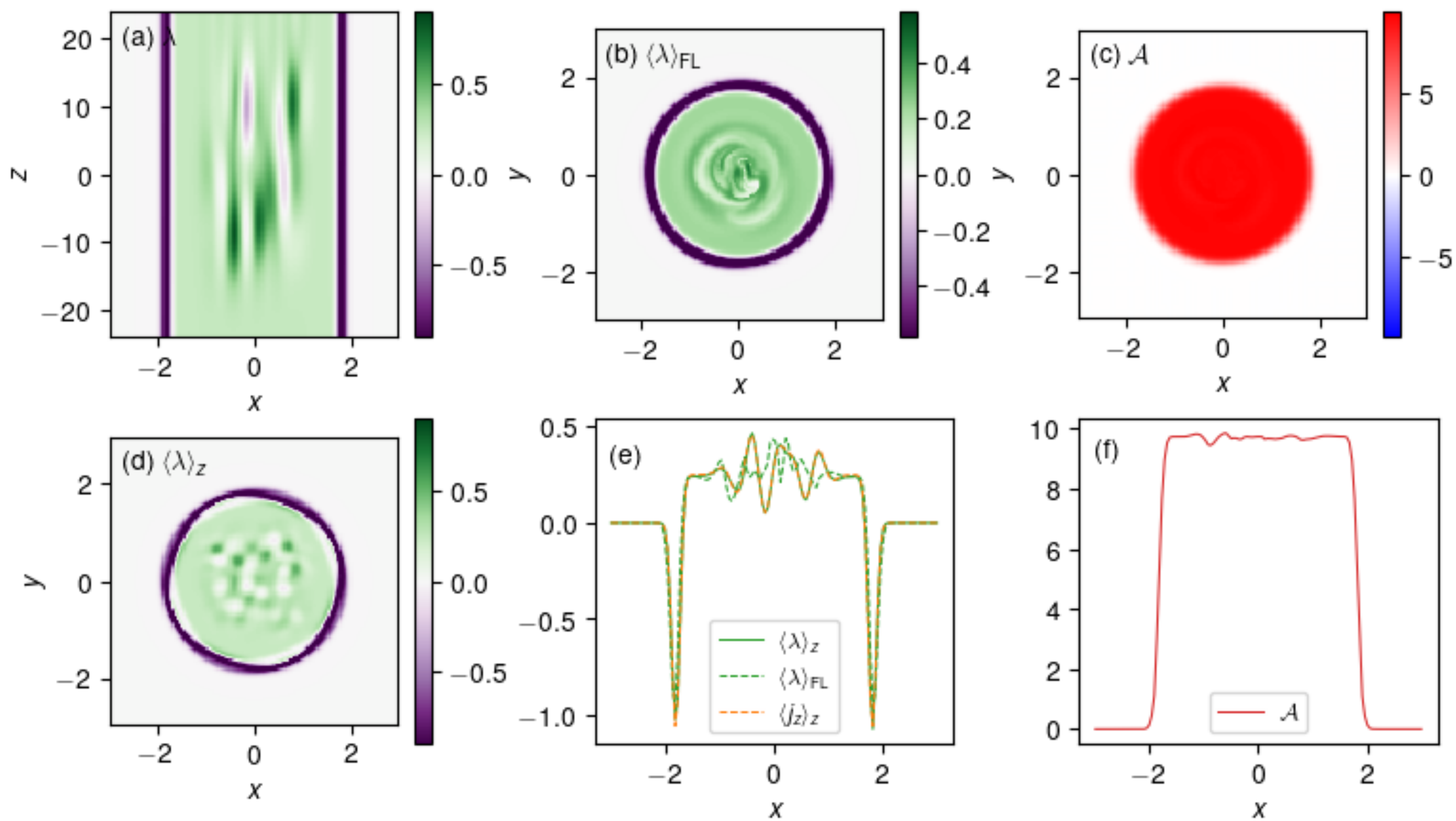
$$\mathcal{A} = \frac{j_z R^2 L}{4}.$$



- In the presence of local fluctuations on a uniform twist tube, field line helicity is more robust than λ because it is a **global average quantity**.

  [cf. Prior & Yeates, *Astrophys J* **787**, 100, 2014]

- Simple model — uniform twist + fluctuations:



Conclusions

- Field line helicities are not arbitrarily changed but are **redistributed** more effectively than increased/decreased.
- **Self-organization** into a relaxed state with two discrete magnetic flux tubes may be predicted from the initial field line helicity distribution (but not from the total helicity).
- Within each tube, the final state is best described as a state of **uniform field line helicity**, independent of Lundquist number. This arises even from an "approximate" Taylor relaxation where the force-free parameter is rather less uniform.

A.J.B. Russell, A.R. Yeates, G. Hornig & A.L. Wilmot-Smith, Evolution of field line helicity during magnetic reconnection, *Phys Plasmas* **22**, 032106, 2015.

A.R. Yeates & A.J.B. Russell, Evolution of field line helicity in magnetic relaxation, *in preparation.*

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