The impact of magnetic topology on plasma dynamics

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with

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Relaxation magnetic therepy [sic] mattress topper

The Parker problem…

Current Sheet Formation in Magnetostatic Equilibria

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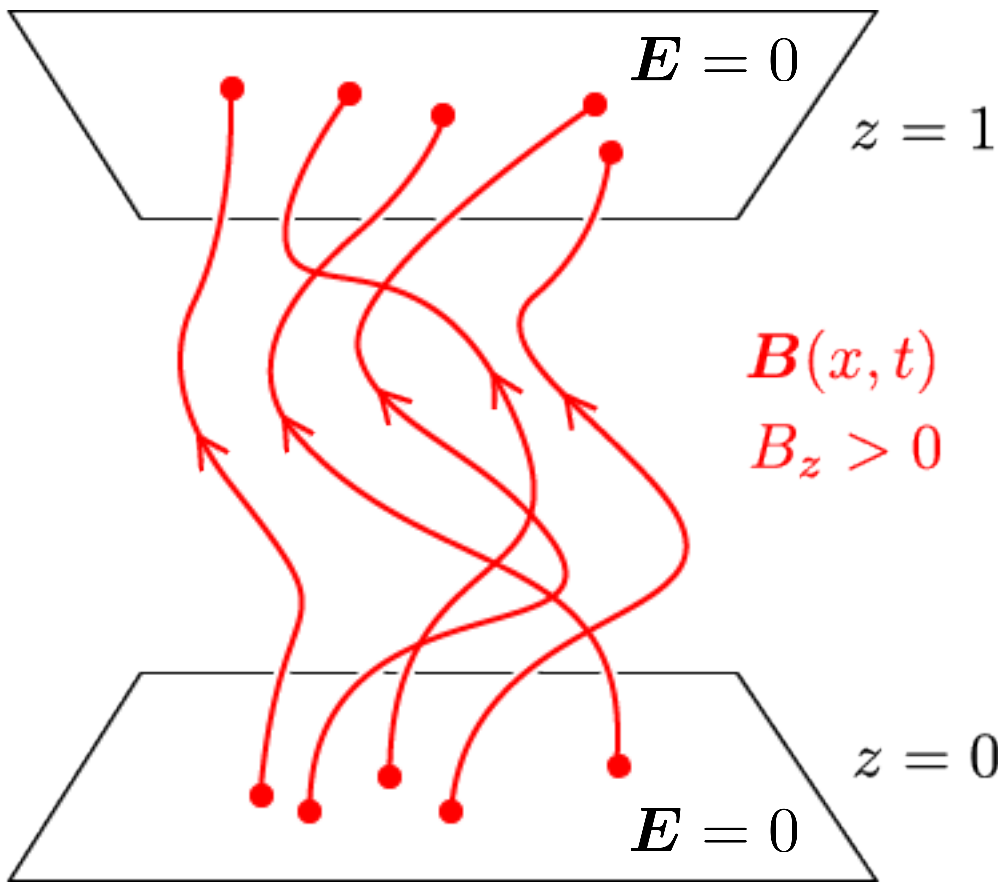
The quasi-static evolution of two-dimensional magnetostatic equilibria is examined in the case where there is a separatrix field line separating regions of different fieldline connectivity. It is shown that in general there will be a current sheet on this separatrix for arbitrarily small displacements of the footpoints. A nonlinear analysis confirms the main results of the linearized theory.

1990

Problem setup

cf. Parker, *ApJ* (1972)

- What is the final state of a **resistive** relaxation with end-points fixed?



$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

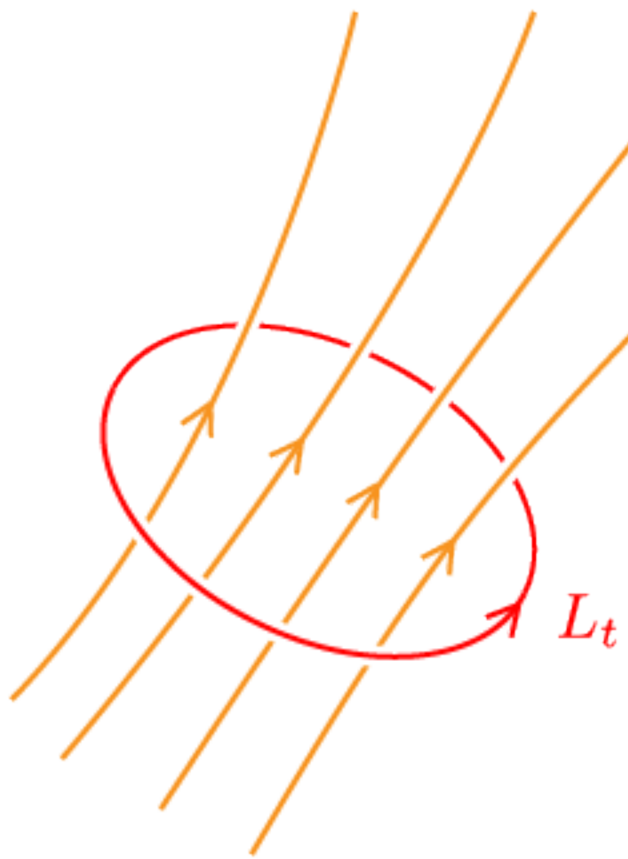
$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{j}$$

- The ultimate end-state is a **potential field** $j = 0$.
- On a dynamical timescale, Taylor (*PRL*, 1974) suggests that we reach a **linear force-free field** $\boldsymbol{j} = \alpha_0 \boldsymbol{B}$, determined by total magnetic helicity.

"An infinity of constraints"

Taylor, *Rev. Mod. Phys.* (1986)

- In **ideal MHD**, the magnetic flux through every closed field line is invariant.

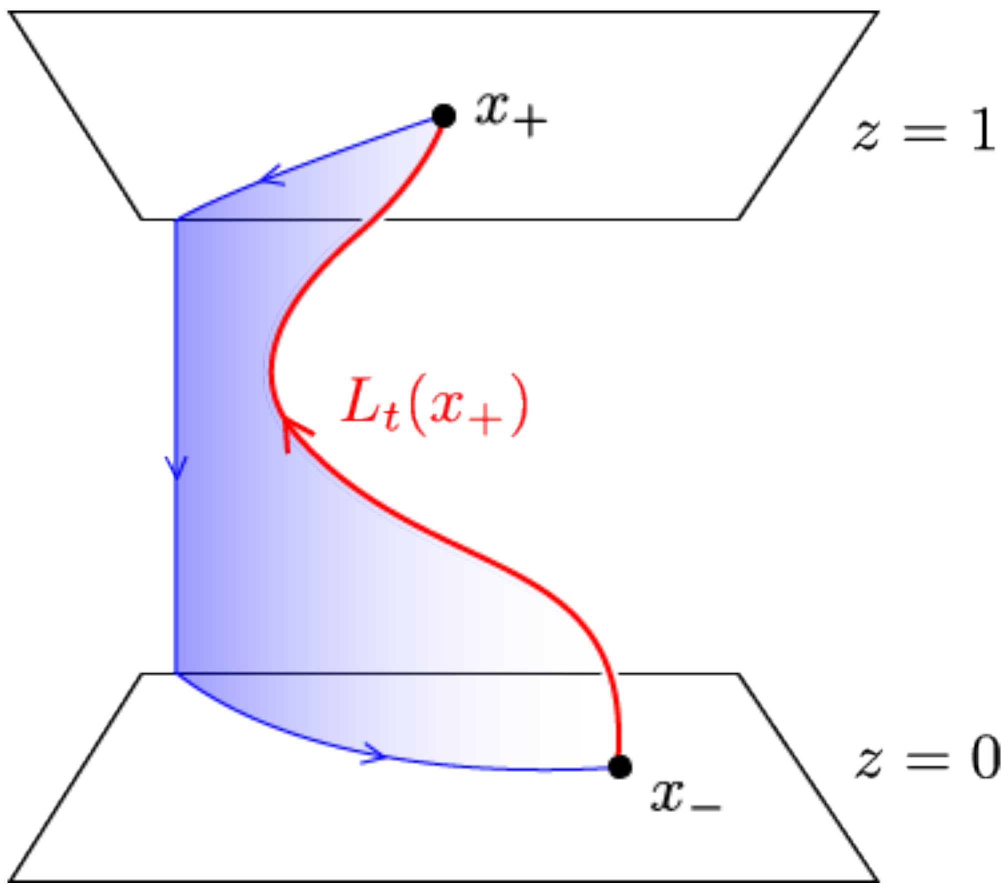


$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{L_t} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l} = 0$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Field line helicity

- To define an ideal invariant flux we complete the loop by a curve on the boundary.



- Whatever the choice of surface, there is a gauge of $\boldsymbol{A}$ in which

$$[\text{flux}] = \int_{L_t(x_+)} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l} := \mathcal{A}(x_+, t)$$

— called **field line helicity**

- The field line helicity is a density for magnetic helicity in this gauge:

$$\int_{z=1} \mathcal{A}(x_+, t) B_z(x_+) \, \mathrm{d}S = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V = H$$

Evolution — 1. Ohm's law

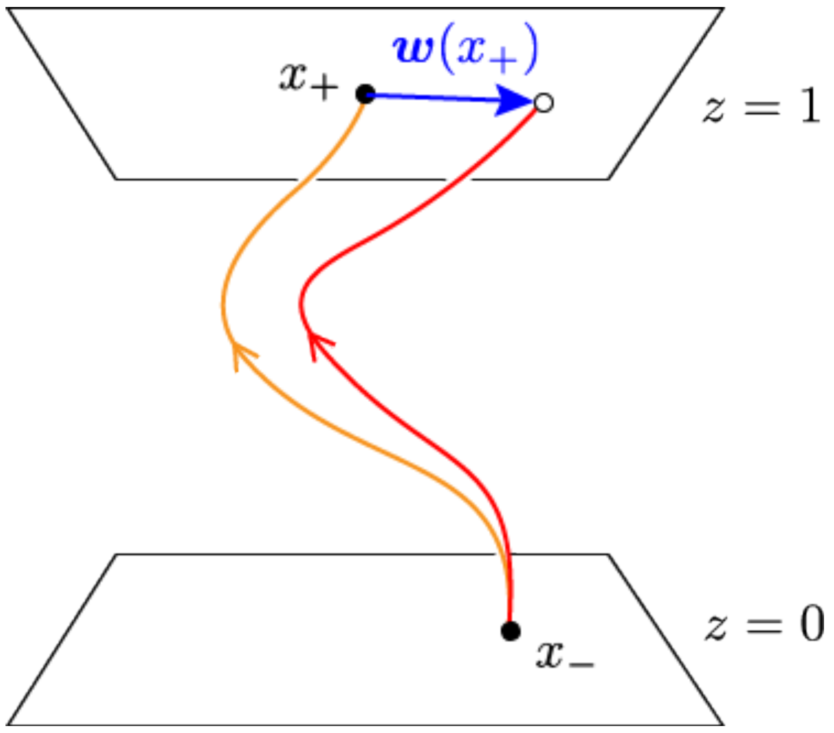
- **Trick:** since $\boldsymbol{B} \neq 0$, recognise that the field lines are frozen-in to a flow $\boldsymbol{w}$.

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{j} \qquad \eta \boldsymbol{j} = -\boldsymbol{u} \times \boldsymbol{B} + \nabla \psi \qquad \psi(x) = \int_{x_-}^{x} \eta \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{l}$$

$$\Longrightarrow \boldsymbol{E} = -\underbrace{(\boldsymbol{v} + \boldsymbol{u})}_{\boldsymbol{w}} \times \boldsymbol{B} + \nabla \psi$$

— so field lines move with the flow $\boldsymbol{w}$ (Newcomb, *Annal. Phys.*, 1958).

- Since $\boldsymbol{E} = 0$ on the boundaries, we have $\boldsymbol{w} = 0$ and $\psi = 0$ on $z = 0$, but not on $z = 1$.
- The parallel component of $\boldsymbol{w}$ is arbitrary, so we can set $w_z = 0$.
- Then $\boldsymbol{w}(x_+)$ tells you how the upper end-points of the field lines are moving due to reconnection.



Evolution — 2. Field line helicity

- Knowing that L_t is frozen-in to the flow $\boldsymbol{w}$, we can calculate

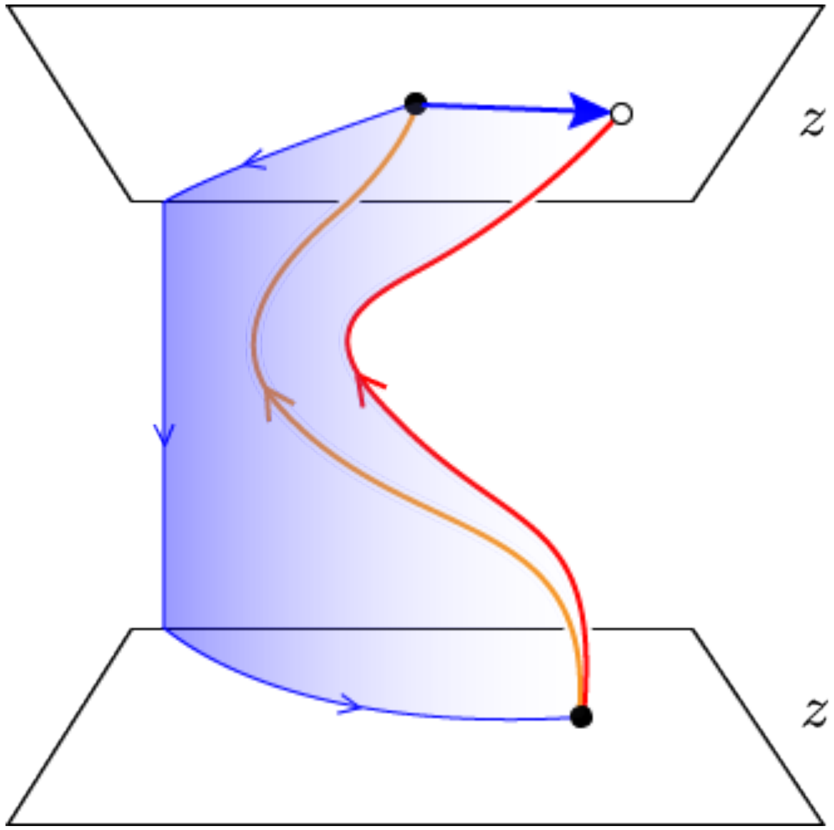
$$\frac{\mathrm{d}\mathcal{A}(x_+,t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{L_t(x_+)} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l} \quad = \int_{L_t(x_+)} \mathrm{d}\boldsymbol{l}\cdot\nabla\left(\boldsymbol{w}\cdot\boldsymbol{A} - \psi - \phi\right)$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = -\boldsymbol{E} - \nabla\phi$$

$$= \boldsymbol{w}(x_+)\cdot\boldsymbol{A}(x_+) - \psi(x_+).$$

- On $z=1$, we have

$$\frac{\partial\mathcal{A}}{\partial t} + \boldsymbol{w}\cdot\nabla\mathcal{A} = \boldsymbol{w}\cdot\boldsymbol{A} - \psi$$



Scaling analysis

$$\frac{\partial \mathcal{A}}{\partial t} + \boldsymbol{w} \cdot \nabla \mathcal{A} = \boldsymbol{w} \cdot \boldsymbol{A} - \psi$$

- Suppose $\boldsymbol{B}$ and $\boldsymbol{E}$ vary on the lengthscale L. So

$$\mathcal{A} \sim LA \qquad\qquad A \sim LB$$

- From Ohm's law,

$$\boldsymbol{w} \times \boldsymbol{B} = \nabla\psi - \boldsymbol{E} \qquad \Longrightarrow \qquad w \sim \frac{1}{\ell B}\psi$$

where ℓ is the lengthscale on which $\nabla\psi$ varies.

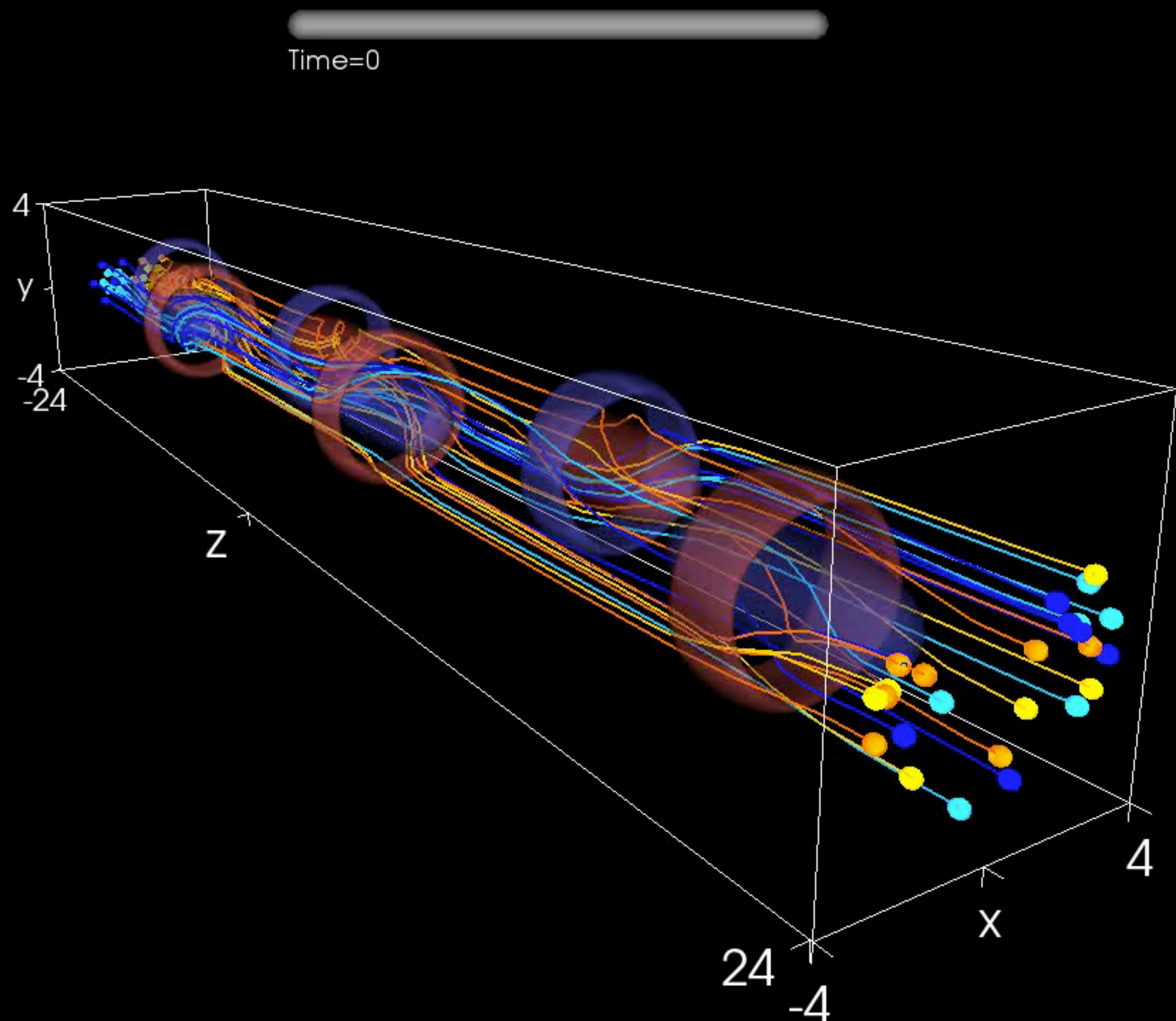
$$|\boldsymbol{w} \cdot \boldsymbol{A}| \sim \frac{L}{\ell}\psi \qquad\qquad |\boldsymbol{w} \cdot \nabla\mathcal{A}| \sim \left(\frac{L}{\ell}\right)^2 \psi$$

In a magnetic field with even moderately complex field line mapping,

$$\psi \ll |\boldsymbol{w} \cdot \boldsymbol{A}| \ll |\boldsymbol{w} \cdot \nabla\mathcal{A}|$$

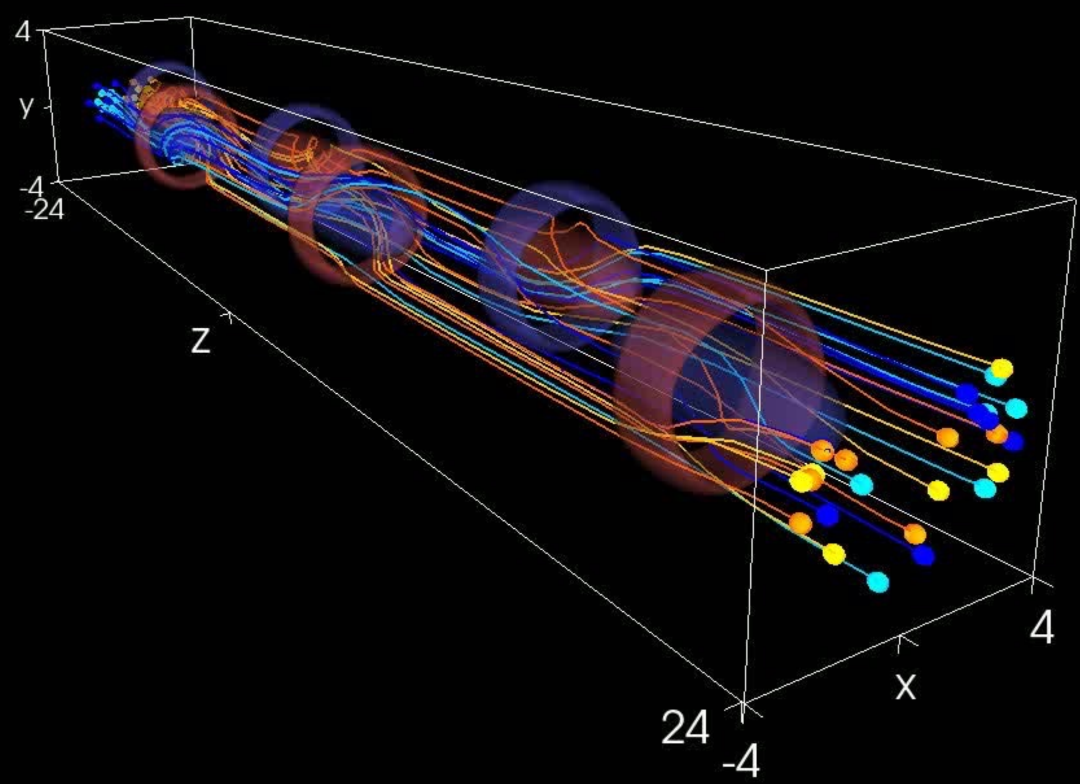
A numerical demonstration

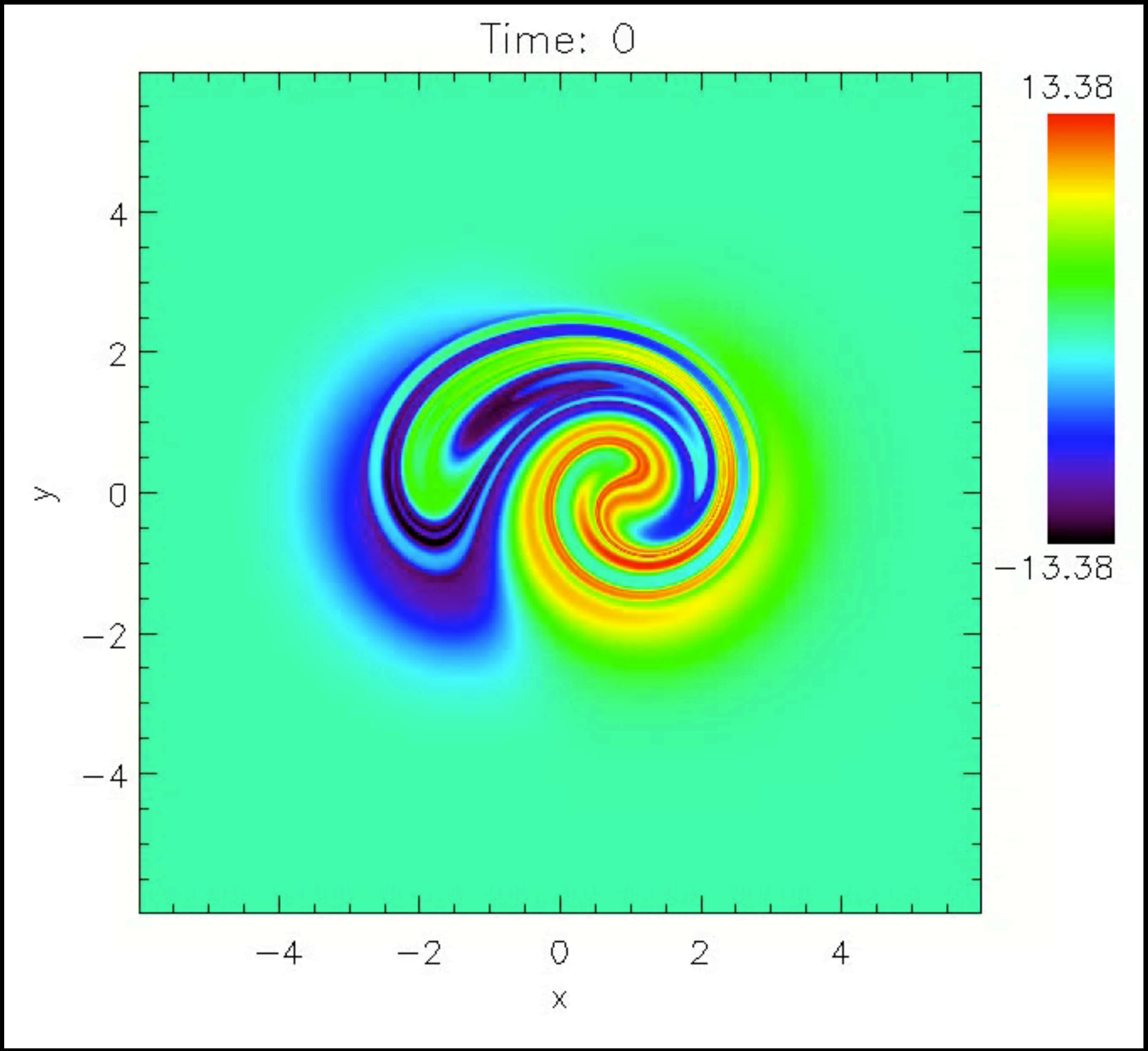
e.g. Pontin et al., *Astron. Astrophys.* (2011)



A numerical demonstration

field line helicity:





Conclusion

- **Total helicity is not the only invariant in a turbulent magnetic relaxation.**

- **In a sufficiently complex field line mapping, field line helicity is efficiently redistributed by reconnection, but not destroyed!**

- Russell, Yeates, Hornig & Wilmot-Smith, "Evolution of field line helicity during magnetic reconnection", *Phys. Plasmas* **22**, 032106 (2015).

- Yeates, Russell & Hornig, "Physical role of topological constraints in localized magnetic relaxation", *Proc. R. Soc. A* **471**, 20150012.

http://www.maths.dur.ac.uk/~bmjg46/

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