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Revisiting Taylor relaxation

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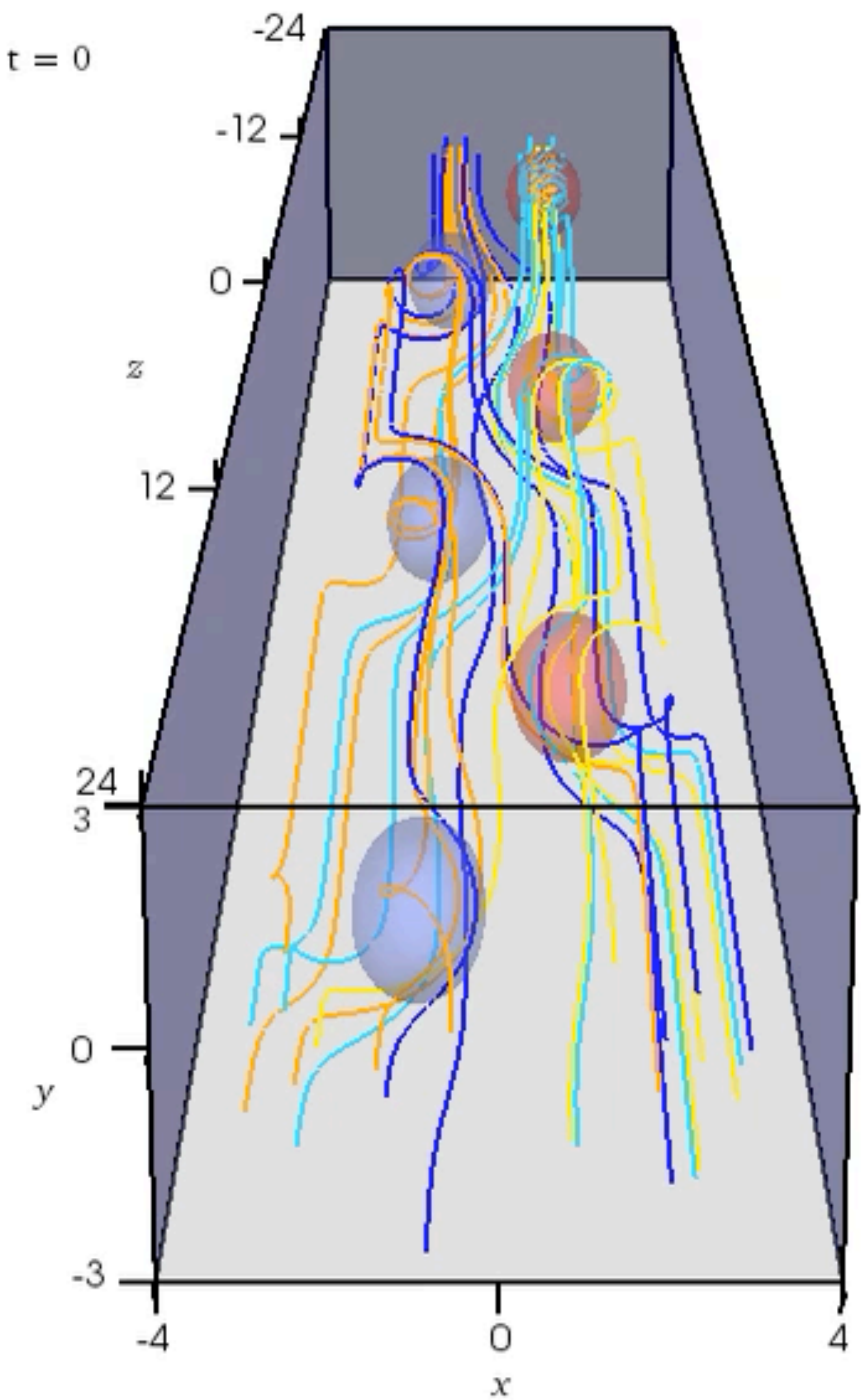
The Dundee braiding experiment

- **Idea**: start with a complex magnetic structure and let it relax under resistive MHD.



[simulations: Lare3D - Arber *et al.*]

- highly "**mixed**"
- **unstable**: launches torsional Alfvén waves generating turbulence with thin current sheets and reconnection
- final state shows **self-organisation** into two oppositely-twisted flux tubes

[**Review:** Pontin et al., *PPCF* **58**, 054008, 2016]

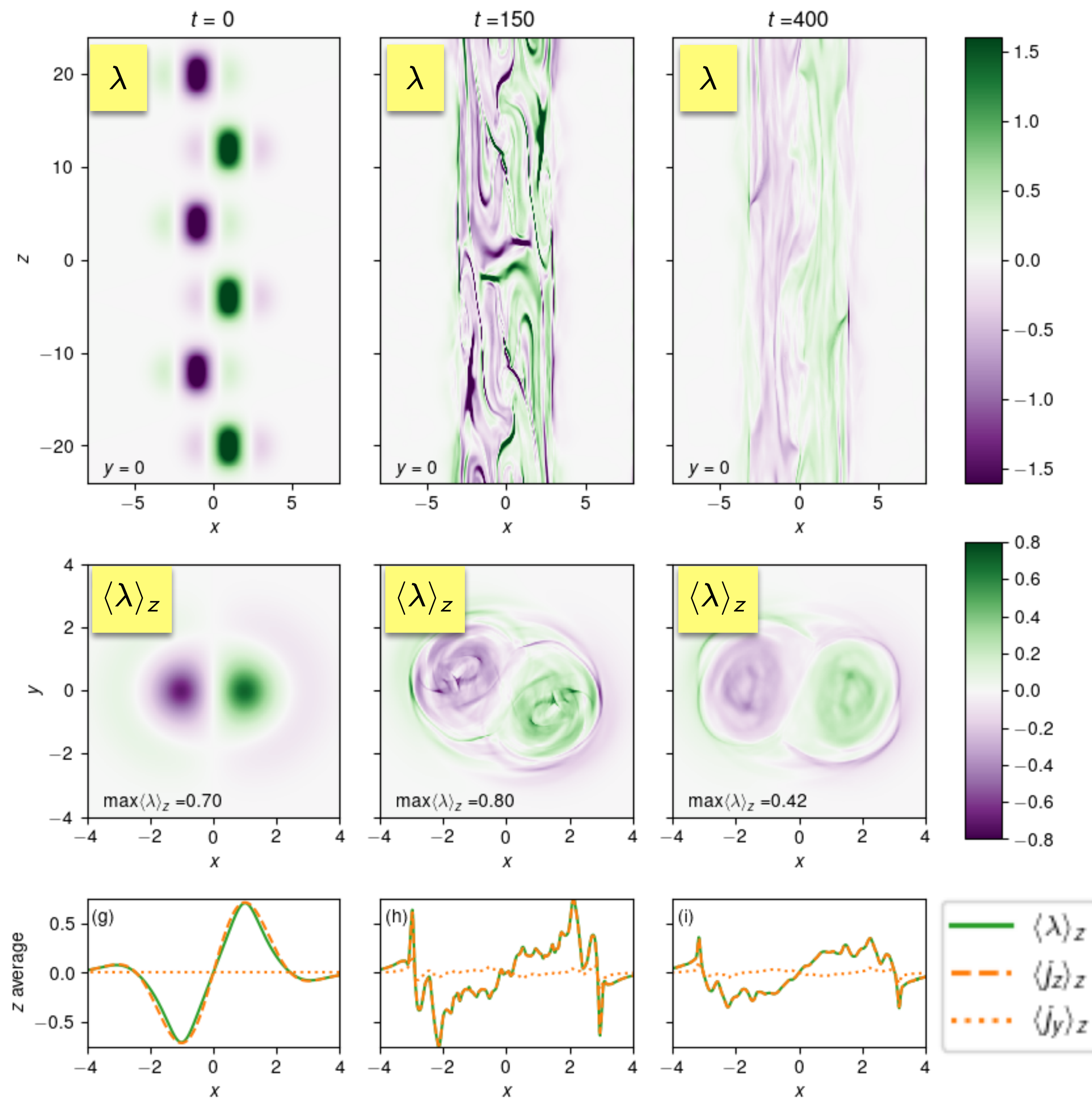
Taylor relaxation

- Classical theory for turbulent magnetic relaxation: assume total (magnetic) helicity is the only invariant, implying a linear force-free final state, $\nabla \times \mathbf{B} = \lambda_0 \mathbf{B}$.

[Taylor, *Rev Mod Phys* **58**, 741, 1986]

- We observe weak flattening, but only within each tube.

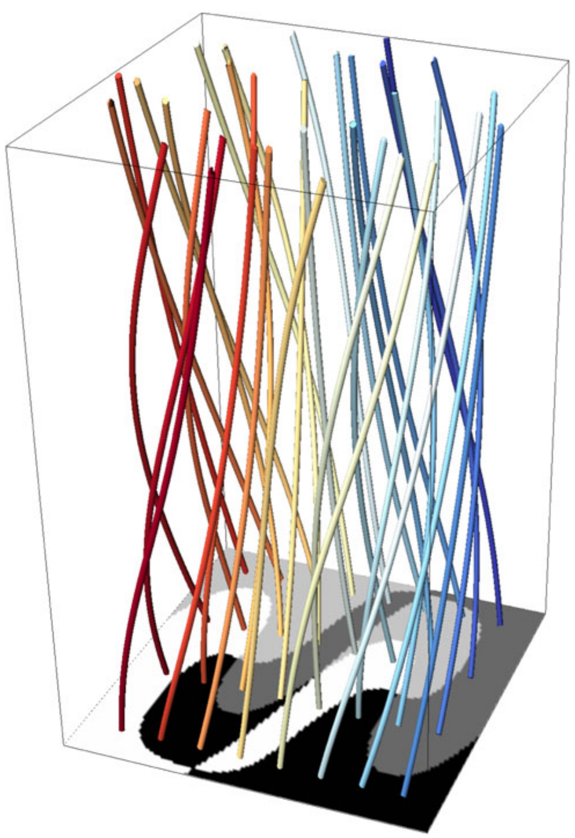
$$\lambda = \frac{\mathbf{j} \cdot \mathbf{B}}{|\mathbf{B}|^2}$$

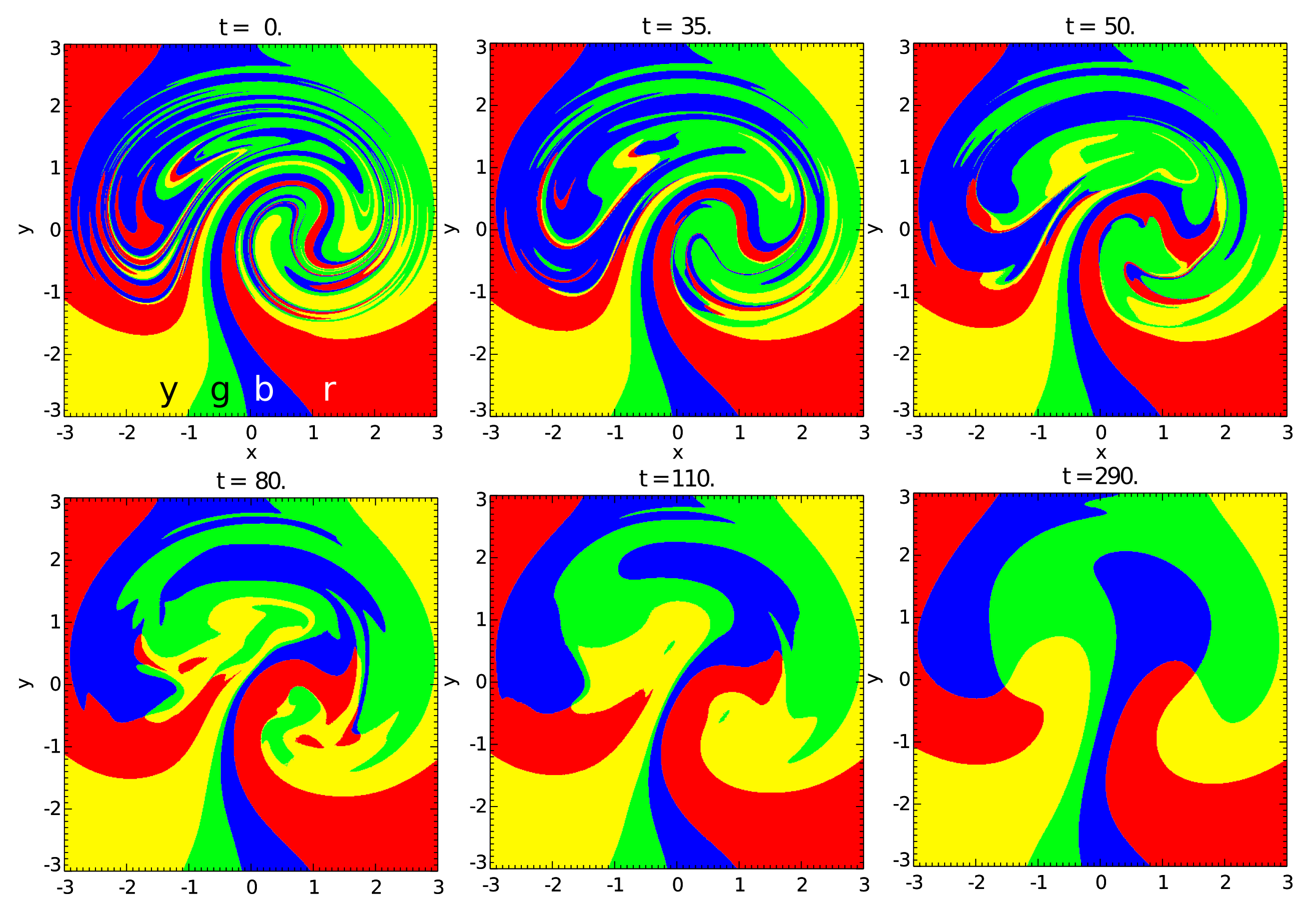


Topological degree

- Preservation of two tubes is a consequence of **spatial localisation** of the dynamics.

- Degree of the field line mapping is determined by its initial structure in the ideal region around the edge where it remains unchanged. [+ continuity]





[Yeates, Hornig & Wilmot-Smith, *PRL* **105**, 085002, 2010;
Yeates, Russell & Hornig, *Proc R Soc* **471**, 20150012, 2015]

Substructure of the tubes?

- **Field line helicity** is a useful measure.

$$\mathcal{A}(L) = \lim_{\epsilon \to 0} \frac{\int_{V_\epsilon(L)} \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V}{\Phi(V_\epsilon(L))} = \int_L \mathbf{A} \cdot \mathrm{d}\mathbf{l}$$

[Berger, *Astron Astrophys* **201**, 355, 1988; Aly, *Fluid Dyn Res* **50**, 011408, 2018]

- For this type of magnetic field, it is a “complete” invariant (same field line mapping iff same FLH).

[Yeates & Hornig, *Phys Plasmas* **20**, 012102, 2013]

- Taylor knew that FLH is an ideal invariant, but conjectured it uninteresting for relaxation because individual values could be changed by reconnection.

- But FLH evolution equation suggests values are primarily redistributed for high Rm.

[Russell et al., *Phys Plasmas,* **22**, 032106, 2015]

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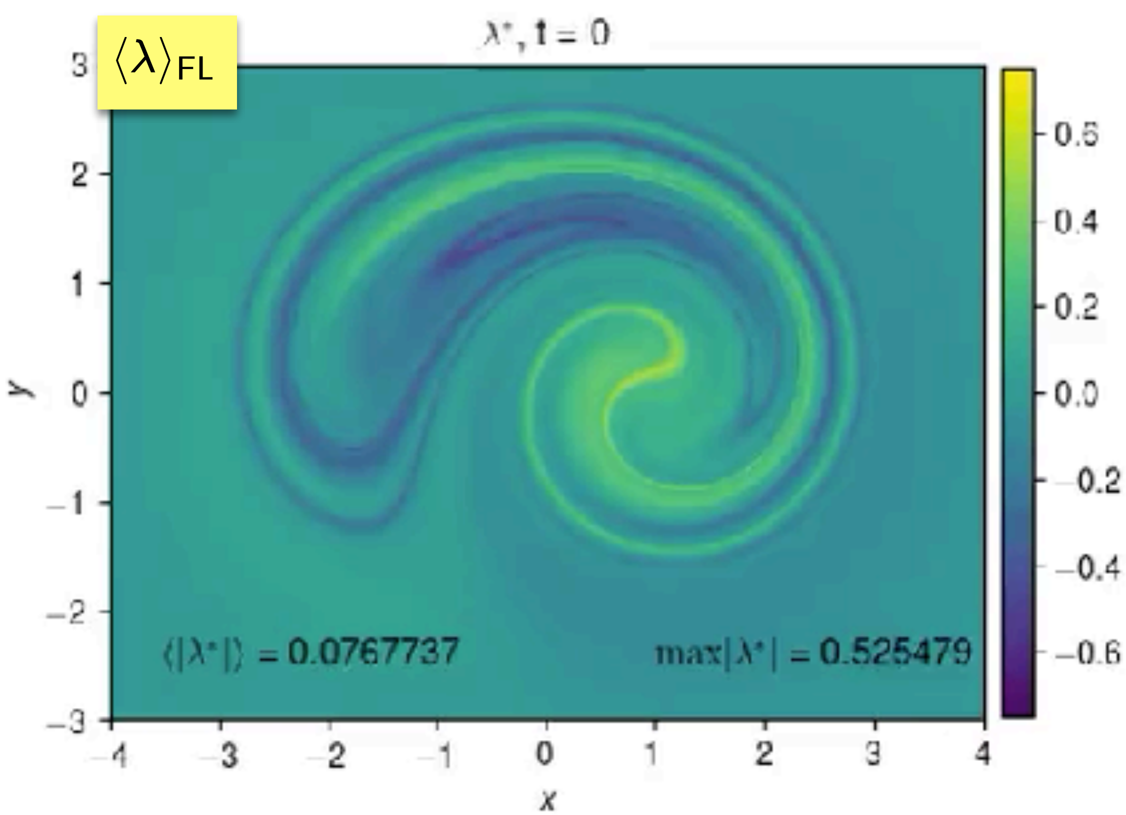
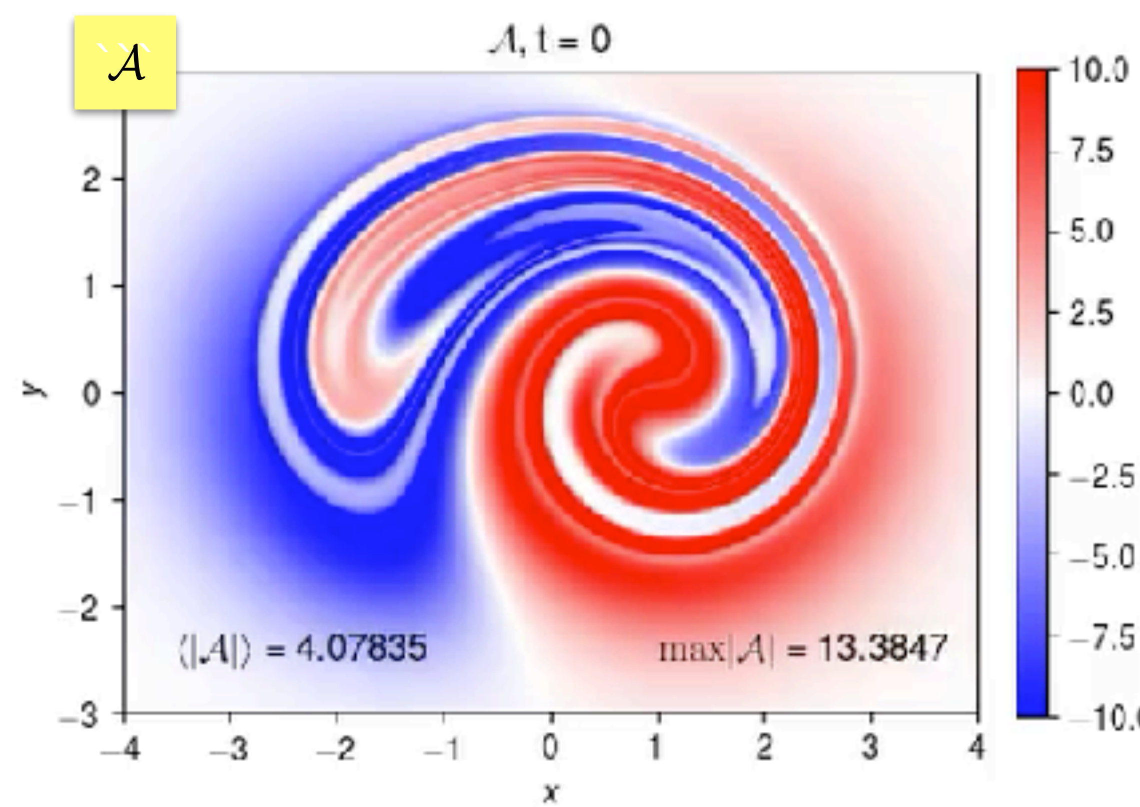
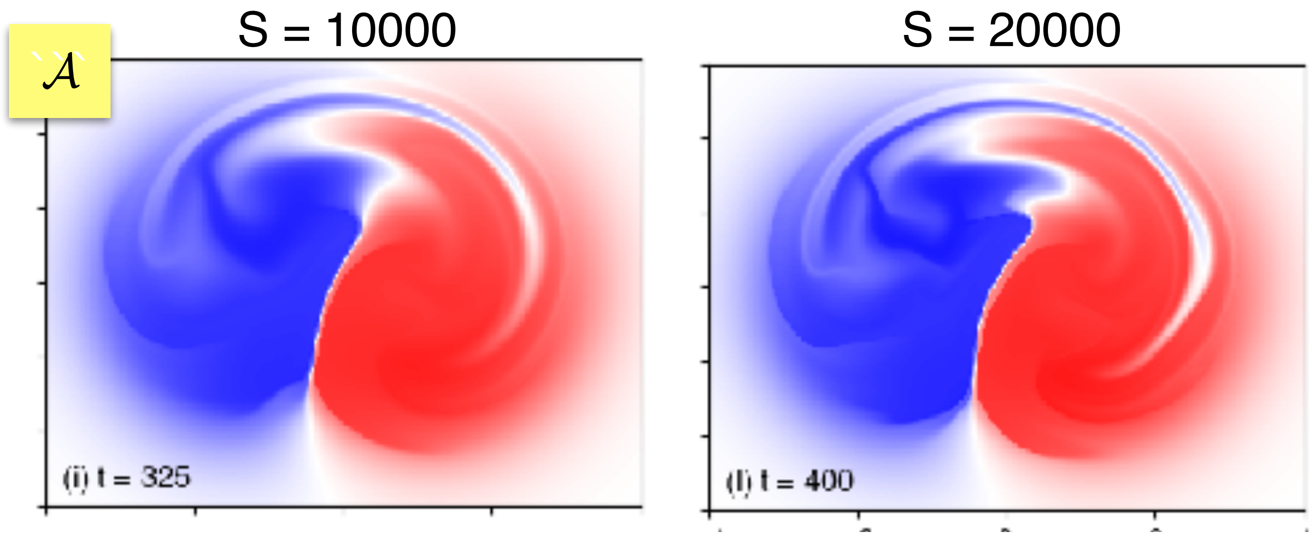
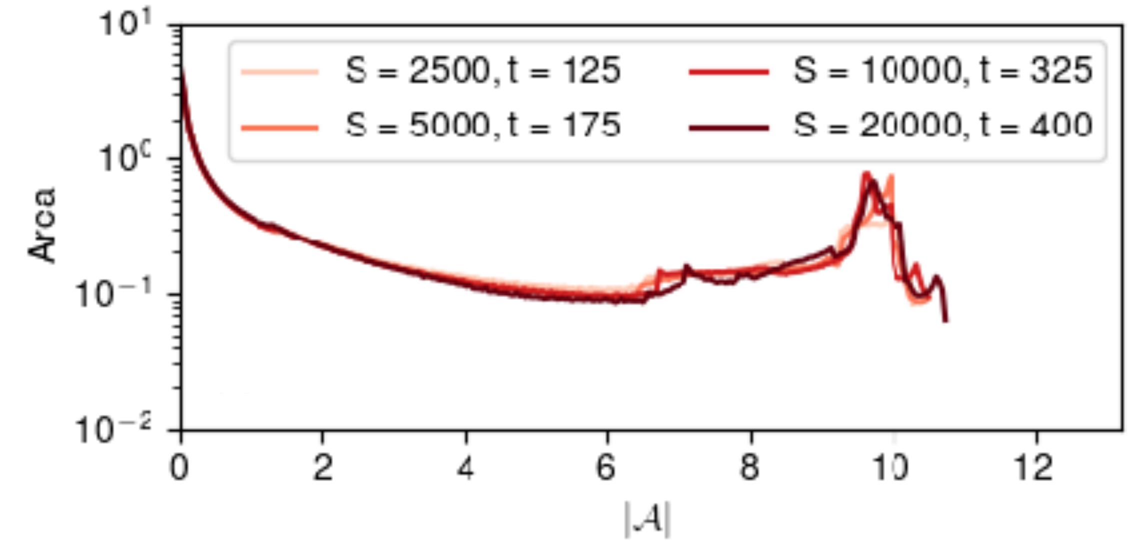




New results

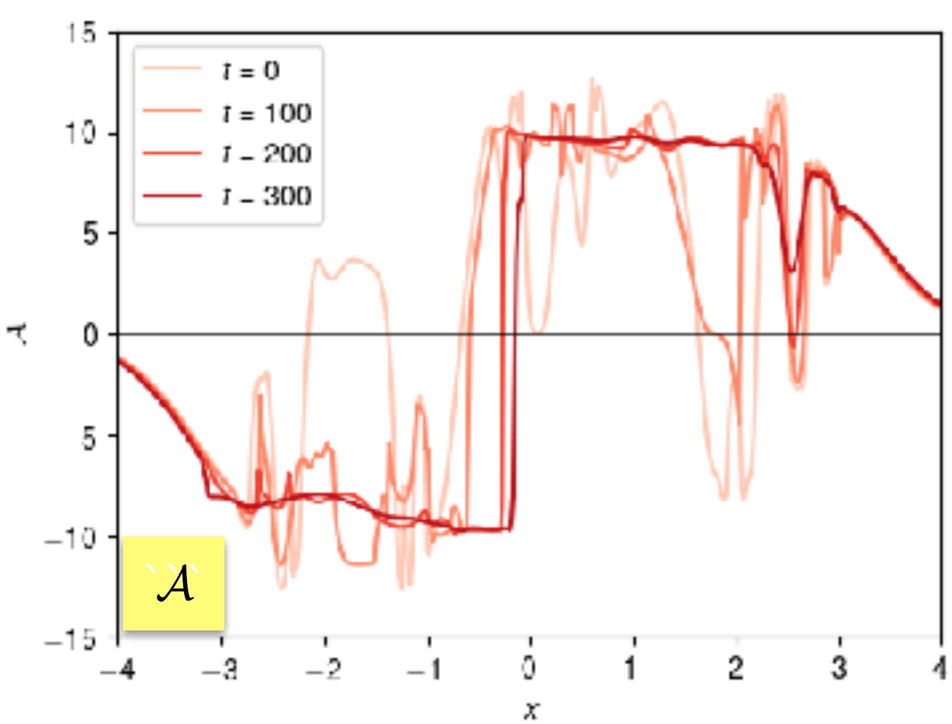
1. Final FLH pattern seems to converge with increasing Lundquist number.





2. FLH is strikingly uniform/flat within each of the final flux tubes.

$y = 0$ cut [S=10000]



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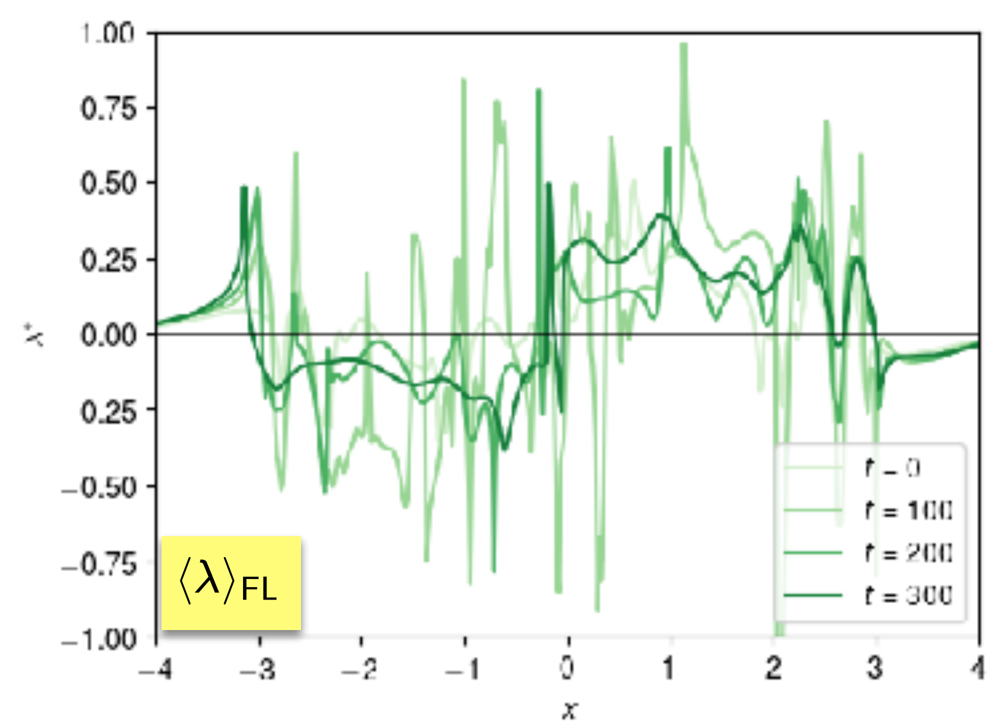
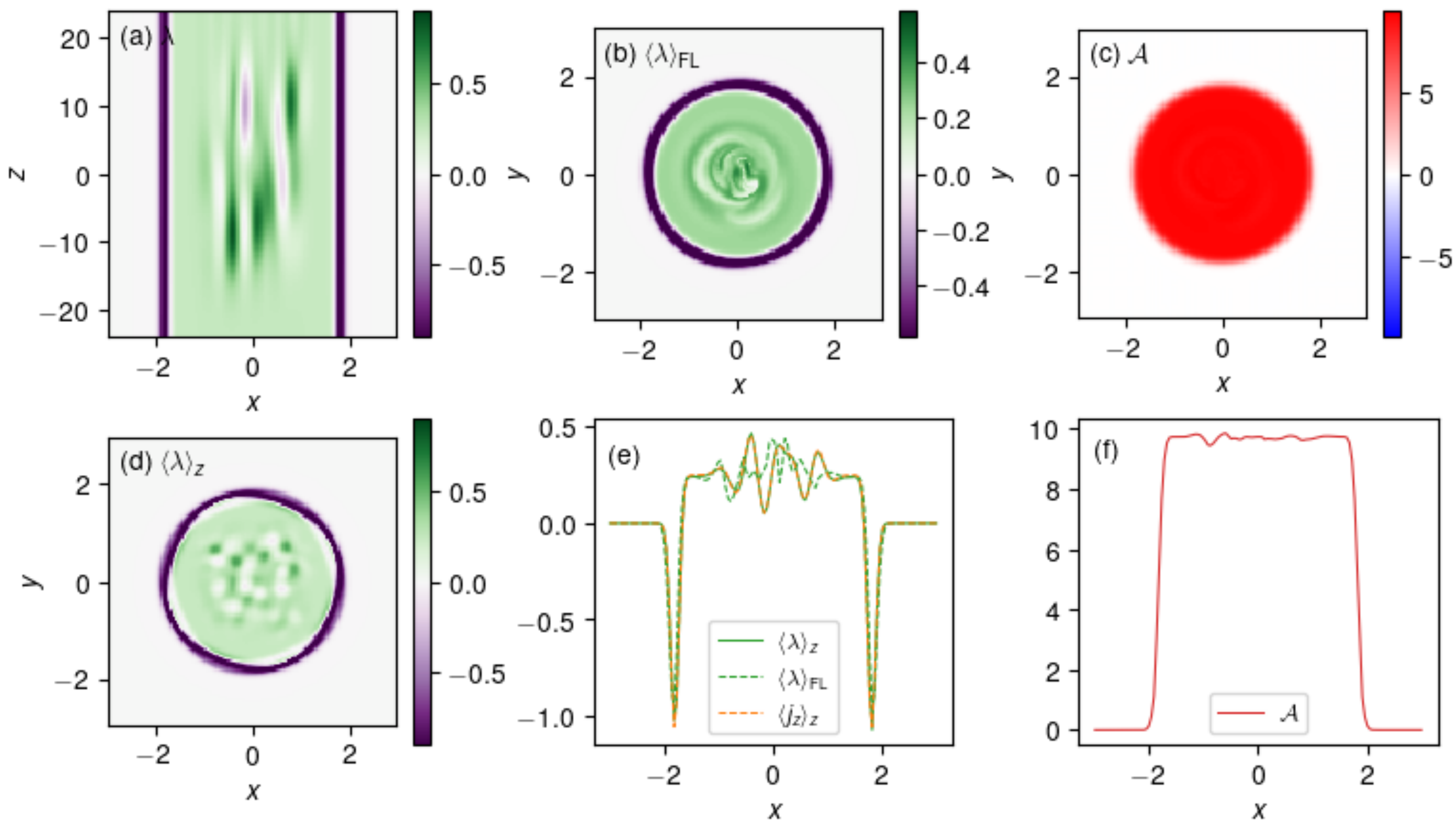


Cause of the flatness?

- **Hypothesis:** uniform FLH is caused by the Taylor relaxation tendency:
  - constant j_z implies uniform-twist field which has constant FLH.
  - FLH is the average winding with all other field lines, so less sensitive to fluctuations.

[cf. Prior & Yeates, *Astrophys J* **787**, 100, 2014]

- e.g. simple toy model (uniform twist + local fluctuations):



Conclusions

- Magnetic braids seem to relax to flux tubes with uniform field line helicity (independent of Lundquist number).

- **Open question**: how general is this behaviour?

A.J.B. Russell, A.R. Yeates, G. Hornig & A.L. Wilmot-Smith, Evolution of field line helicity during magnetic reconnection, *Phys Plasmas* **22**, 032106, 2015.

A.R. Yeates, A.J.B. Russell & G. Hornig, Evolution of field line helicity in magnetic relaxation, *in preparation.*

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